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Supporting the Professional Learning of School Leaders and Teachers

Introduction to Applied Mathematics

Applied Mathematics

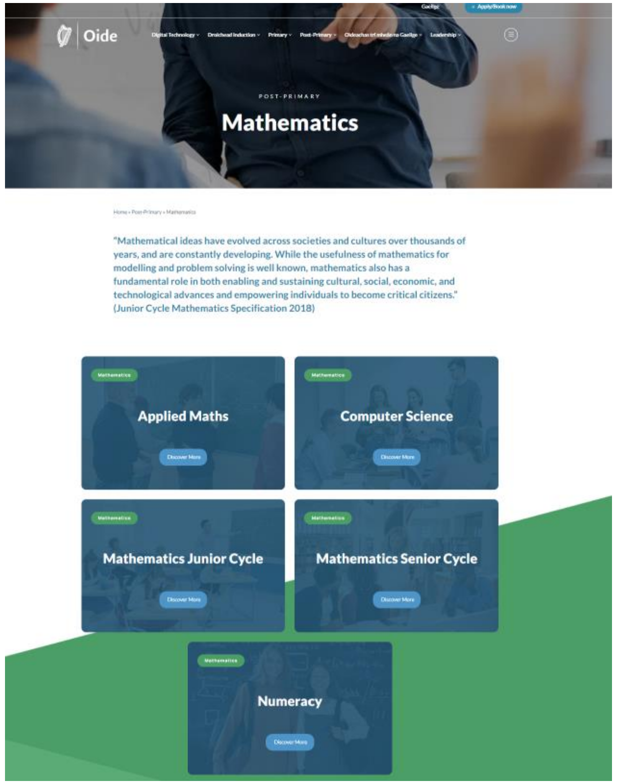
Professional Learning Event (PLE)

Applied Mathematics, Mathematics & Computer Science

https://oide.ie/post-primary/home/mathematics/

- Applied Mathematics
- Mathematics
- Computer Science
- Numeracy

@OideAppliedMath



Applied Mathematics

Oide

Enquiries:
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@OideAppliedMath

Administrator:
Grainne Haughney

Join our mailing list:

SCAN ME

Schedule

09:30 - 11:00	Welcome and Introductions A concepts through Modelling approach to Algorithms
11:00 - 11:30	Tea and Coffee
11:30 - 13:15	Mathematical Modelling Two-Particle Kinematics Problems
13:15 - 14:15	Lunch
14:15 – 16:00	Exploring Differential Equations through the lens of Mathematical Modelling

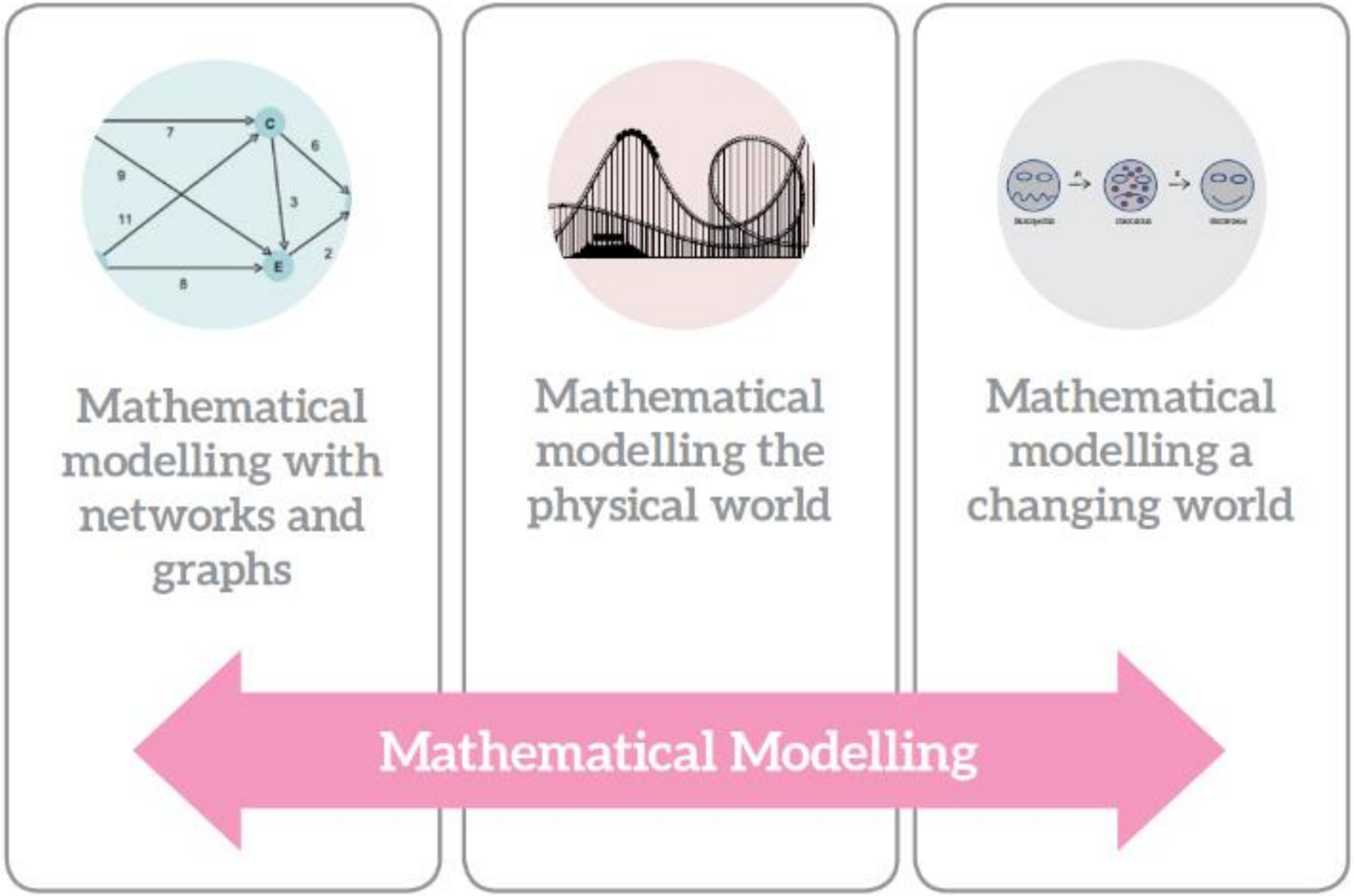
Key Messages

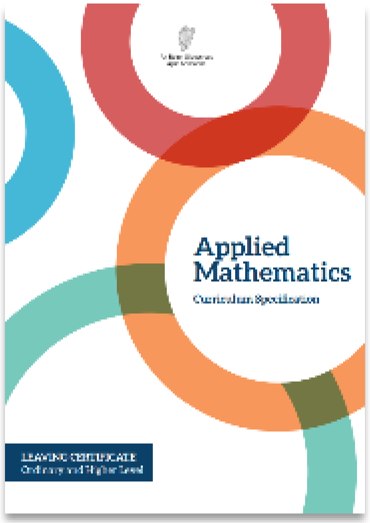
Core to the specification is a non-linear approach empowered by the use of rich pedagogy which promotes the making of connections between various Applied Mathematics learning outcomes.

Strand 1 is the unifying strand and emphasises the importance of utilising mathematical modelling across all learning outcomes.

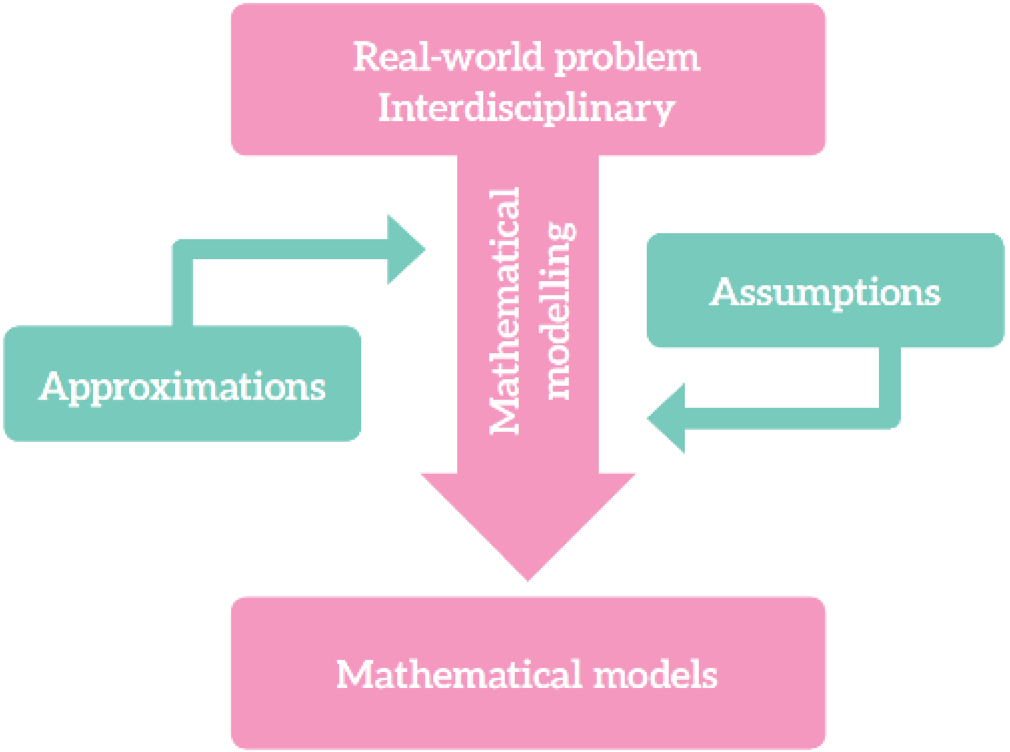
Applied Mathematics is rooted in authentic problems as a context for learning about the application of Mathematics to design solutions for real-world problems and to develop problem solving skills applicable to a variety of disciplines.

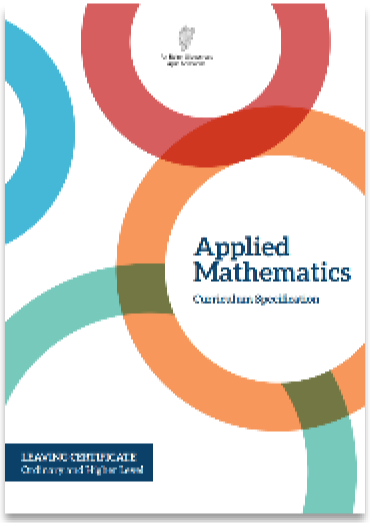
Structure of the Specification





Overview of the Unifying Strand





Assessment & Coursework

Written Assessment
80%

Modelling Project
20%

Ordinary and Higher

Approaches To Mathematical Modelling in The Classroom

1

Concepts then Modelling

Explore a number of mathematical concepts through suitable tasks, word problems etc., then solve a rich modelling problem. In exploring these tasks, modelling competencies may also be developed.

- Complete a full modelling cycle
- Focus on a subset of competencies

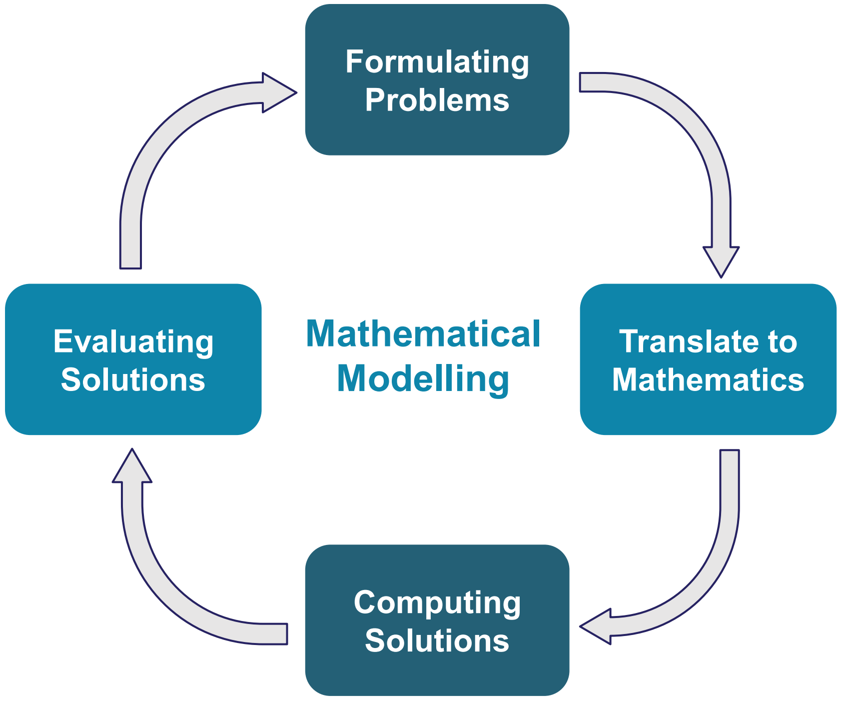
2

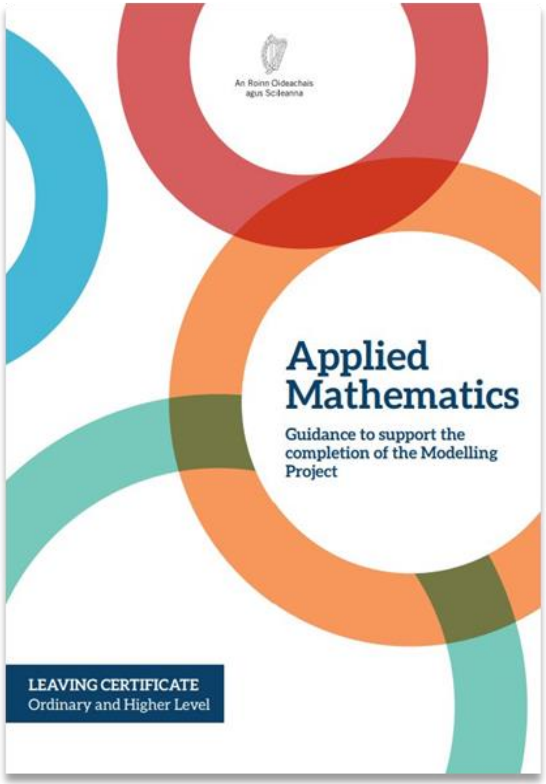
Concepts through Modelling

Explore a rich modelling problem and, as the need arises, develop understanding of new mathematical concepts through instruction, guided discovery, research, etc.

- Complete a full modelling cycle
- Focus on a subset of competencies

Mathematical Modelling Cycle





Senior Cycle Vision & Applied Mathematics

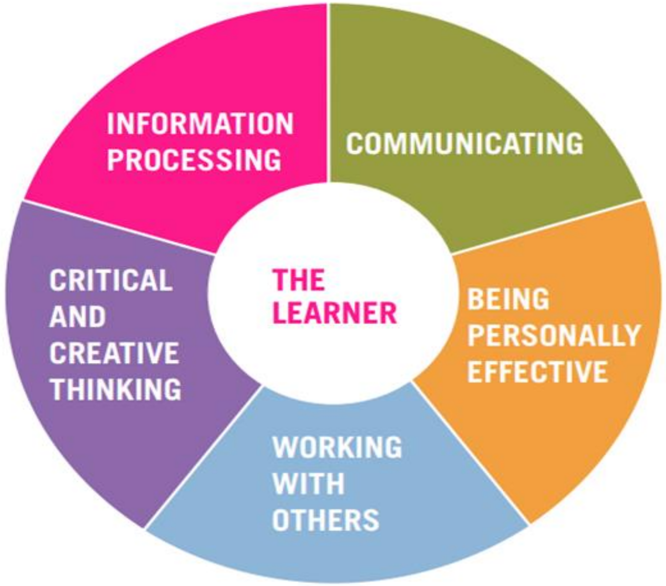
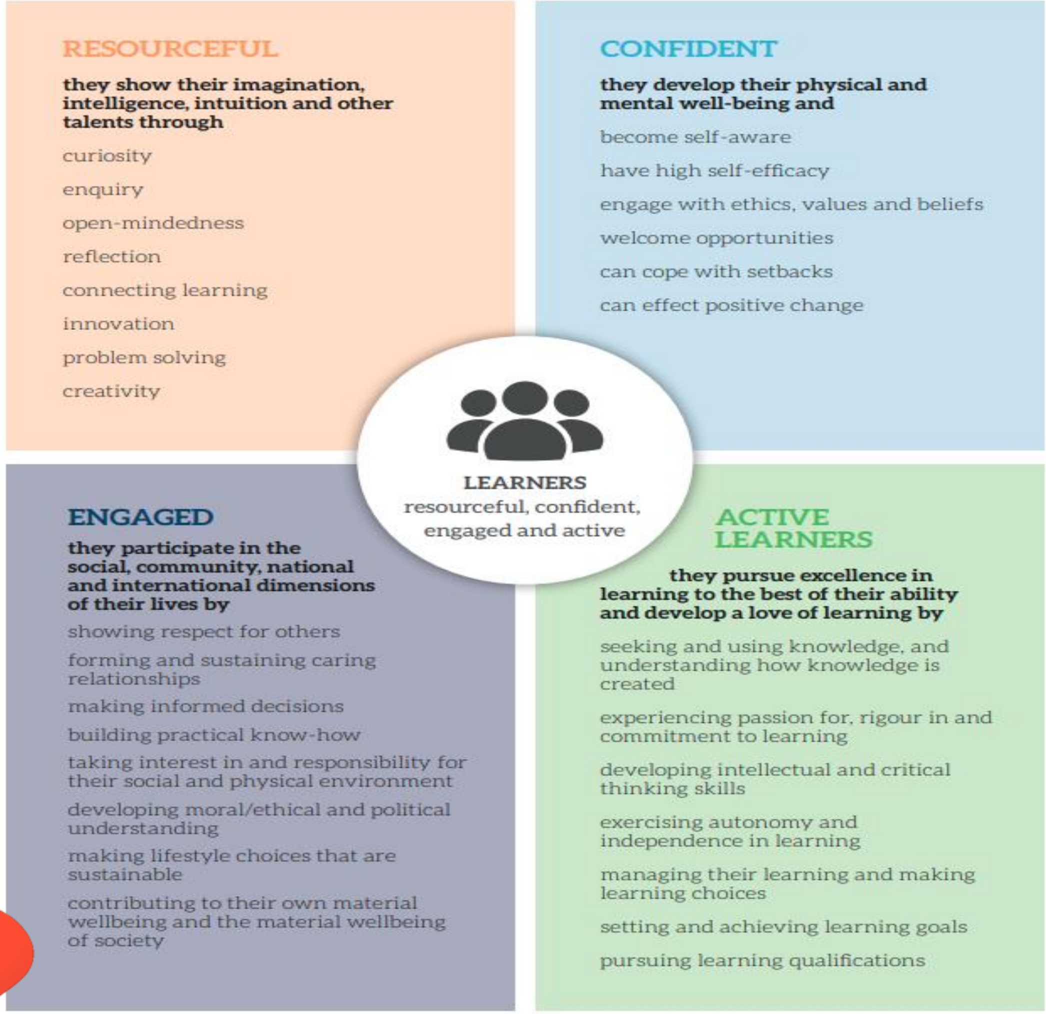


Figure 5: Key skills of senior cycle



What are the benefits of studying Applied Mathematics and how does this fit in to the overall Senior Cycle vision and development of key skills?

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Supporting the Professional Learning of School Leaders and Teachers

Session 1

A Concepts through Modelling Approach to Algorithms

By The End of This Session You Will Have:

Experienced a constructivist approach to learning and teaching algorithms

Engaged in the four stages of the modelling cycle to develop students' understanding of Prim's, Kruskal's and Dijkstra's algorithms

Explored the use of algorithms to solve authentic real-world problems

Made distinctions between the three algorithms and their applications

Interpreting a Real-World Problem

2

Concepts through Modelling

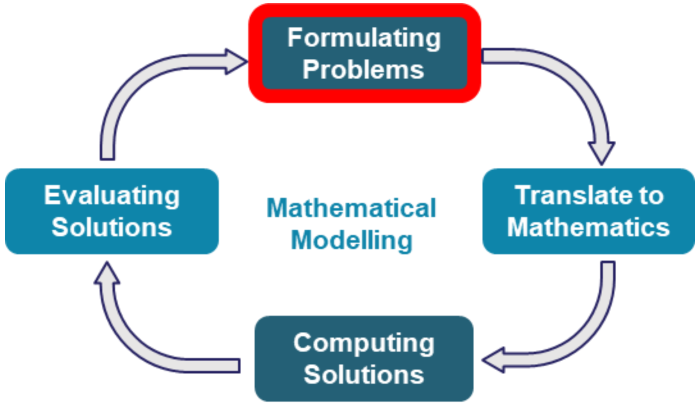
"determine what assumptions are necessary to simplify the problem situation" Specification p. 16

Problem Statement:

Aoife, an Irish fashion designer, based in Dublin is looking to expand her business in Europe.

She plans to visit four of the top fashion capitals – London, Milan, Berlin & Paris.

Aoife will start in Dublin and visit each city. What route should she take, in order to minimise travel time between cities?



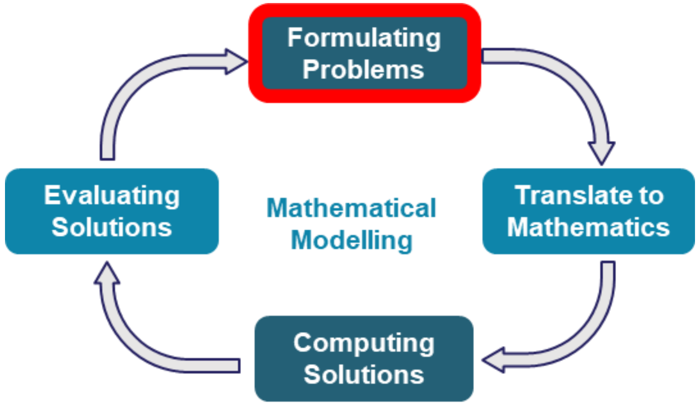
Flying around Europe

2

Concepts through Modelling

“determine what assumptions are necessary to simplify the problem situation” Specification p. 16

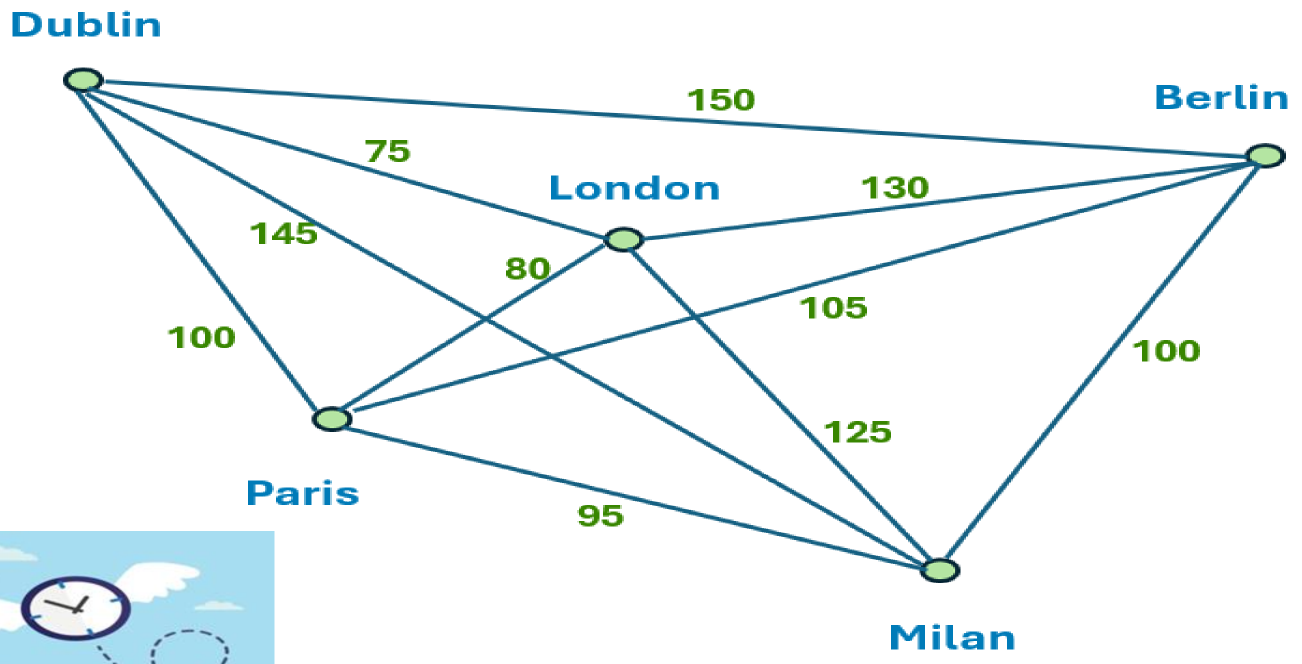
Time (mins)	Dublin	Berlin	Milan	London	Paris
Dublin	-	150	145	75	100
Berlin	150	-	100	130	105
Milan	145	100	-	125	95
London	75	130	125	-	80
Paris	100	105	95	80	-



Flying around Europe

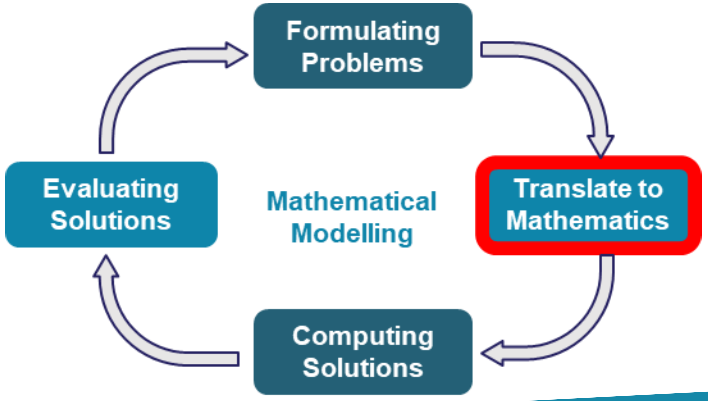
2

Concepts through Modelling



“translate the information given in the problem together with the assumptions into a mathematical model that can be solved” Specification p. 16

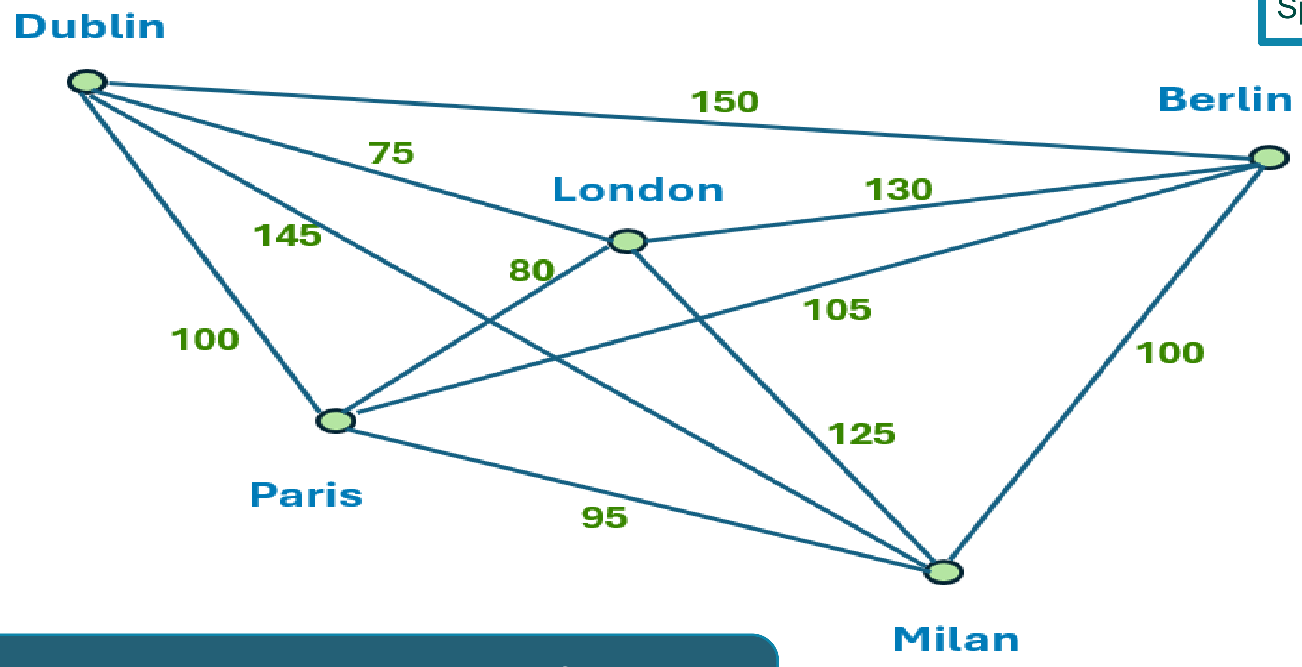
“represent real-world situations in the form of a network” Specification p. 17



Flying around Europe

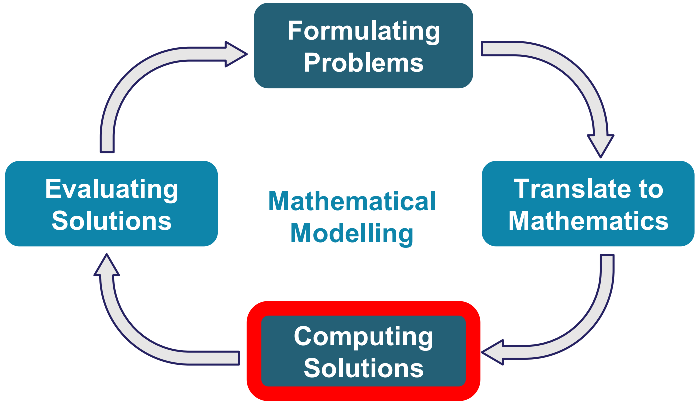
2

Concepts through Modelling



"compute a solution using appropriate mathematics" Specification p. 16

"use appropriate algorithms to find minimum spanning trees" Specification p. 17



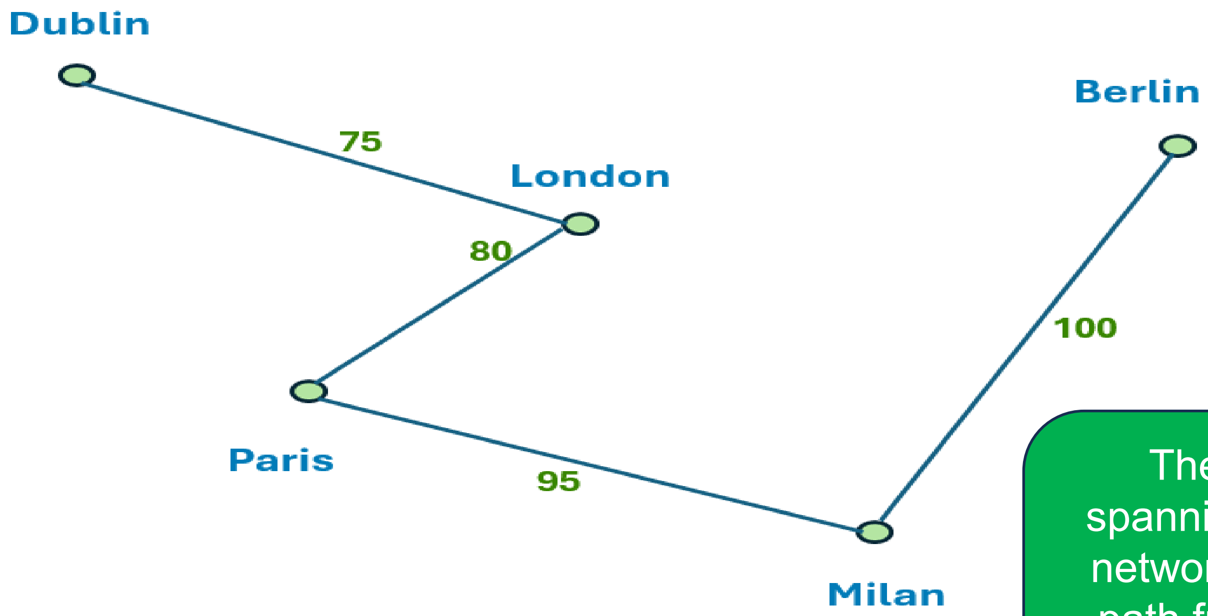
What is the minimum spanning tree for the network?

Flying around Europe

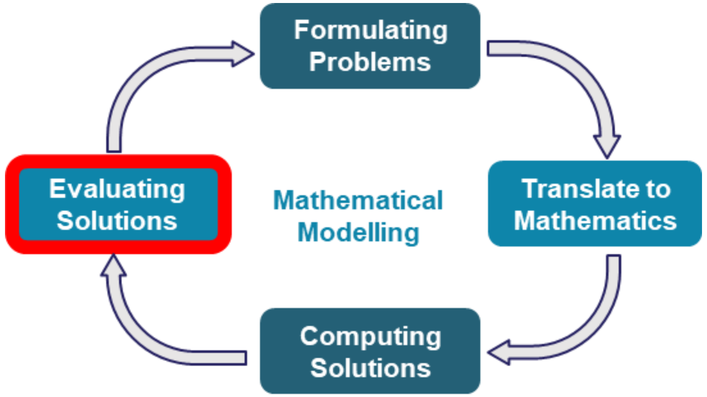
2

Concepts through Modelling

“refine a model and use it to predict a better solution to the problem; iterate the process ” Specification p. 16



The minimum spanning tree of this network produced a path from Dublin to Berlin. Will this always happen?



What are the limitations to this model?
How could we refine the model?

Describe the Best Approach

Considering the approach you used to create the minimum spanning tree (MST) for the flights network, **create** a step-by-step guide for creating a minimum spanning tree for any network

How might your students describe their approach?

Describe the Best Approach

Kruskal's Algorithm

1. Select the edge of least weight
2. Find the next edge of least weight. If it would form a cycle with the edges already selected, don't choose it. If not then add it to the MST
3. If there is a choice of equal edges, it has no effect which you choose first
4. Repeat step 2 until all vertices are connected

Prim's Algorithm

1. Pick any vertex/node (unless a predetermined one is indicated)
2. Find all the edges that connect the tree to new nodes, select the minimum and add it to the tree, ensuring to avoid cycles
3. Keep repeating step 2 until we get a minimum spanning tree with all nodes connected and cycles avoided

"Use algorithms to solve problems "
Specification p. 17

Which algorithm should I use?

"Justify the use of algorithms in terms of correctness " Specification p. 17

Prim's Algorithm	Kruskal's Algorithm
Starts from a single vertex and adds edges one at a time	Sorts edges by weight and adds them to the tree if they don't create a cycle
Generally faster for dense graphs	Works well with sparse graphs, does not require a starting vertex

Interpreting a Real-World Problem

2

Concepts through Modelling

Problem Statement:

On her trip around Europe, the Dublin fashion designer arrives in London Heathrow airport to travel to their next destination.

On arrival, Aoife is made aware that her flight is cancelled. The only two options are to book another flight from Heathrow tomorrow morning or travel to London Stanstead airport to make a flight taking off in 4 hours.

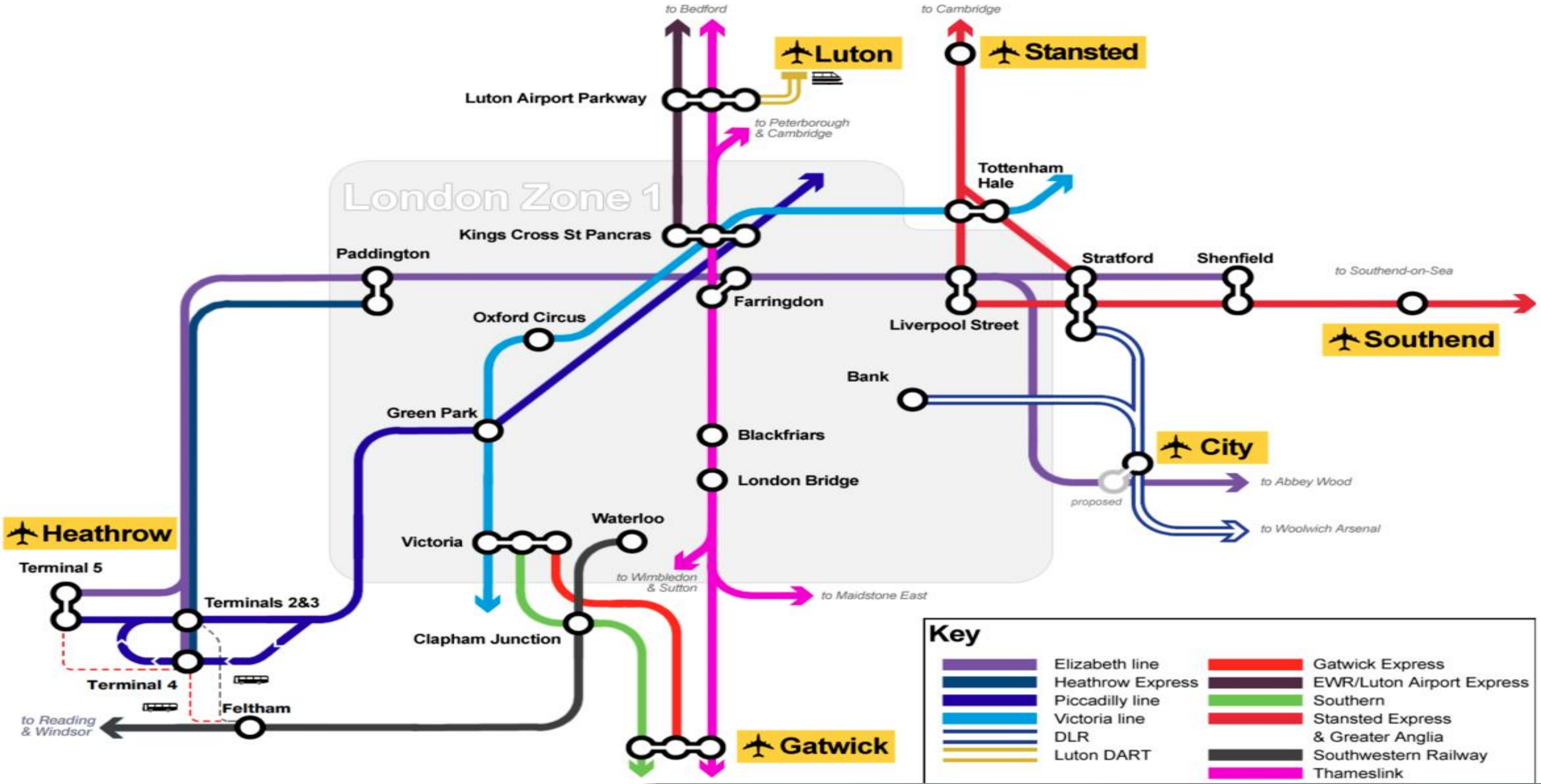
The fashion designer decides to make the trip to Stansted. What is the optimal route?

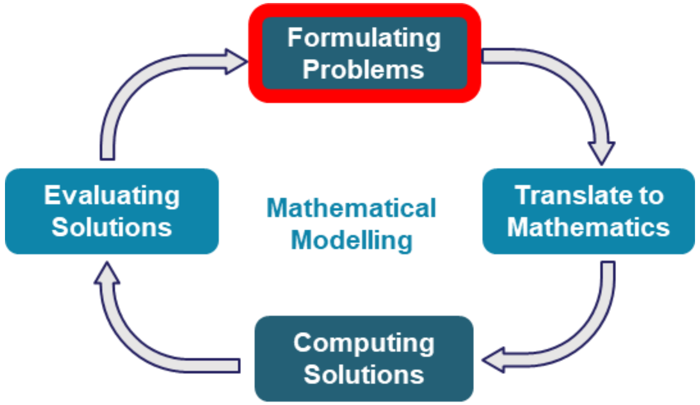
Heathrow to Stansted

2

Concepts through Modelling

“determine what assumptions are necessary to simplify the problem situation” Specification p. 16

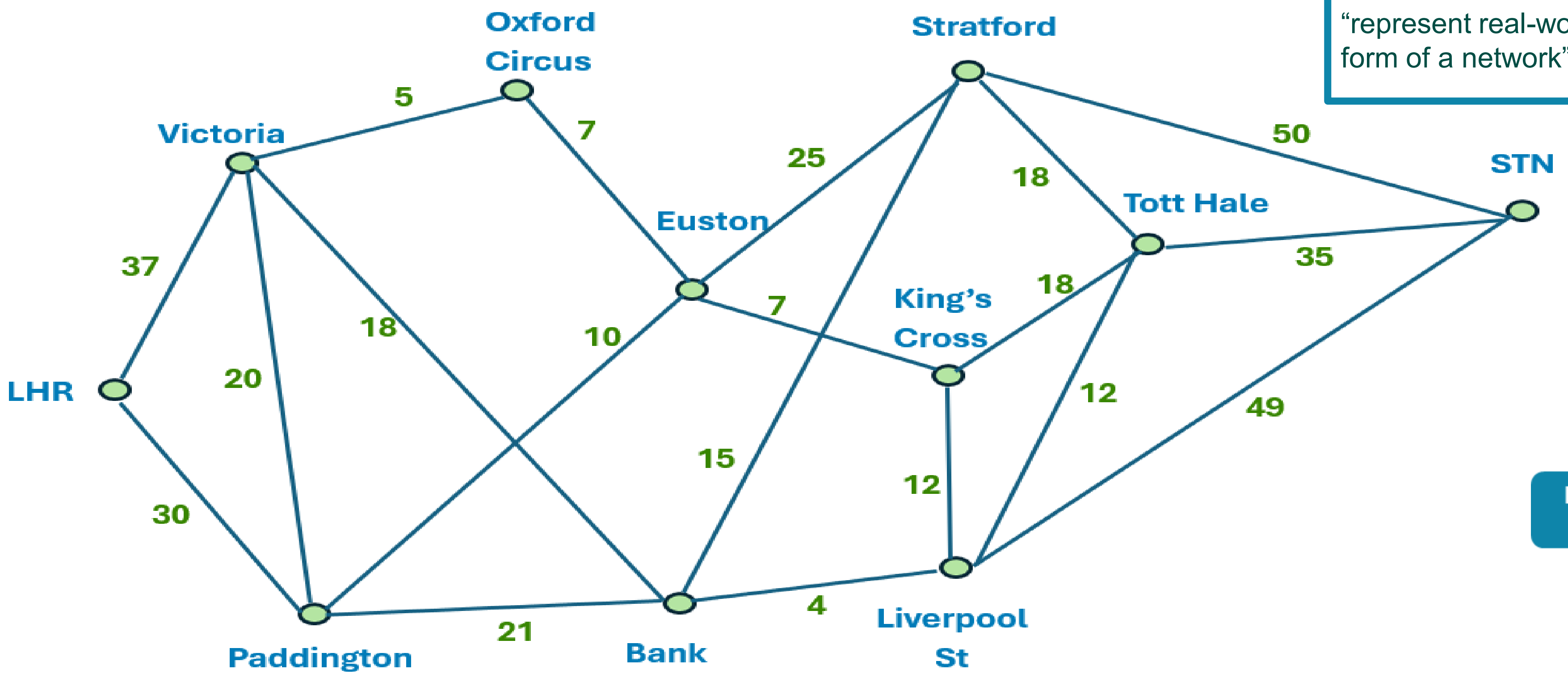




Heathrow to Stansted

2

Concepts through Modelling



"translate the information given in the problem together with the assumptions into a mathematical model that can be solved" Specification p. 16

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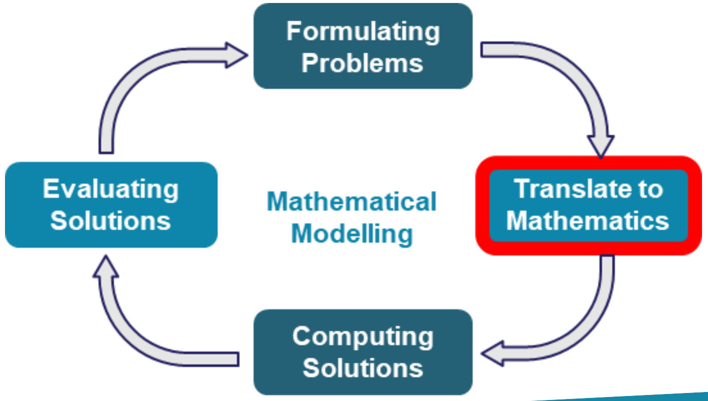
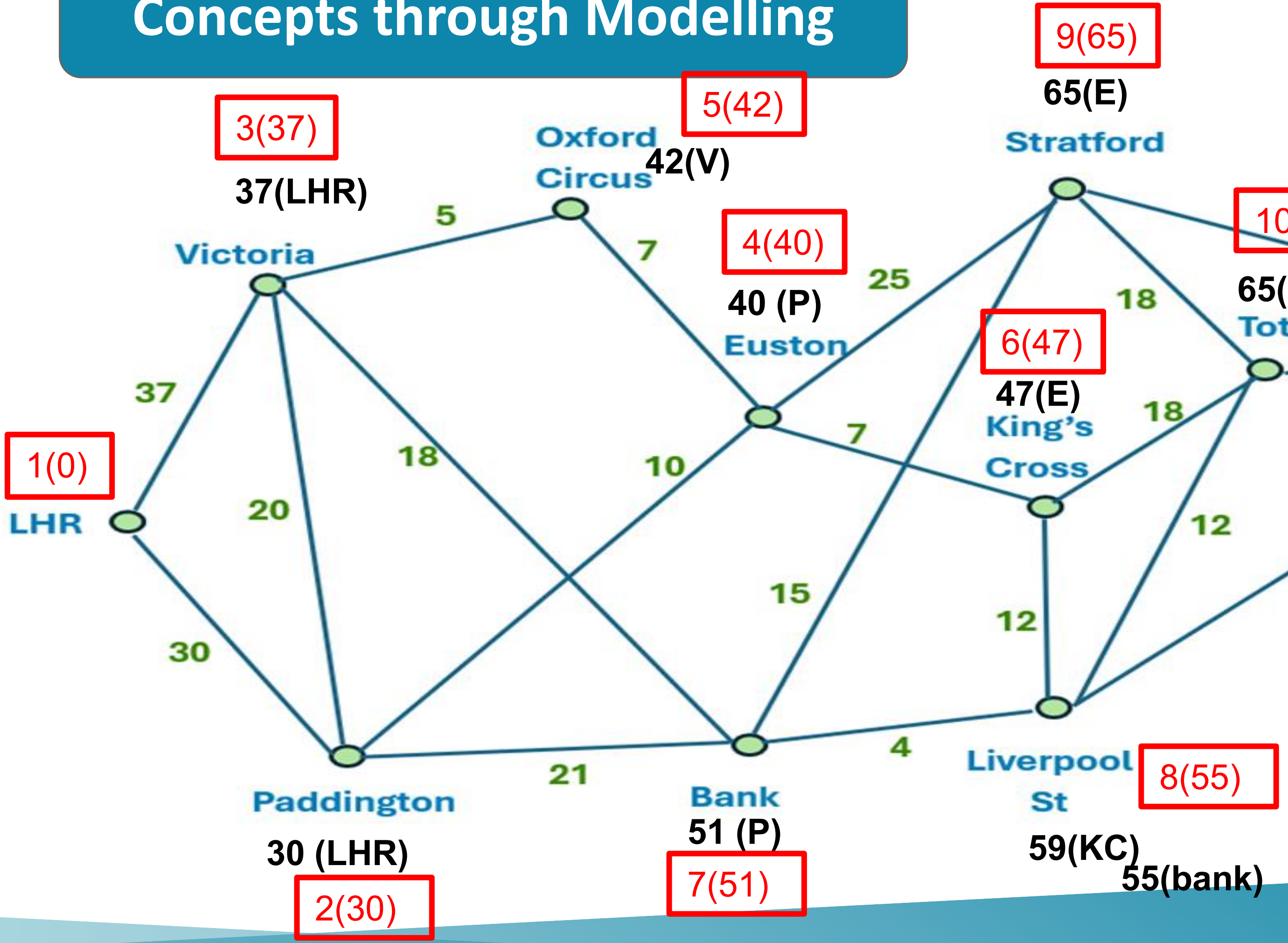


Heathrow to Stansted

2

Concepts through Modelling



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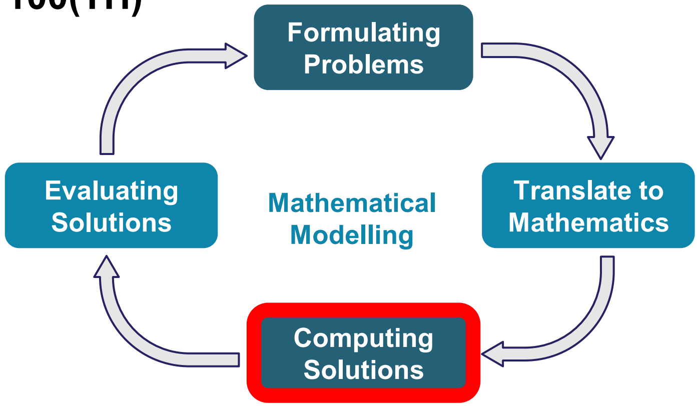
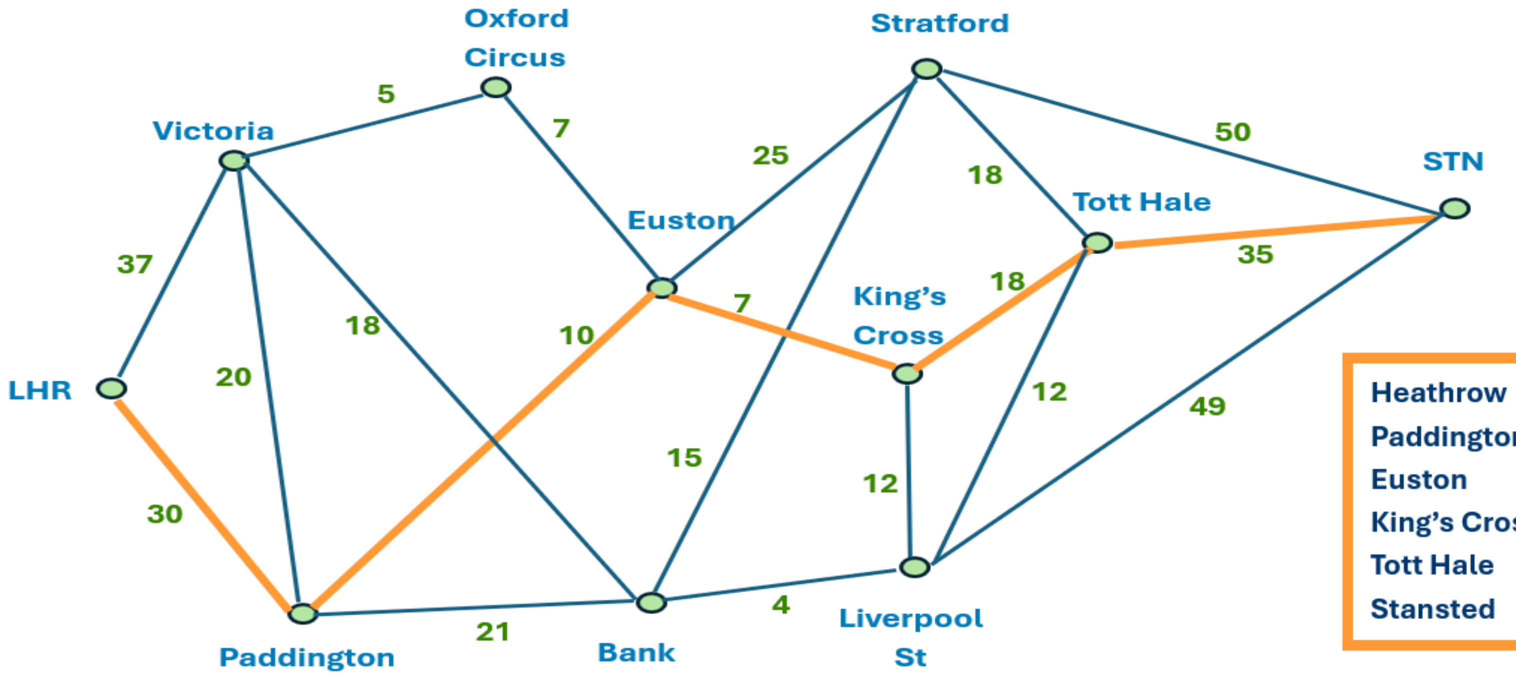


Heathrow to Stansted

2

Concepts through Modelling

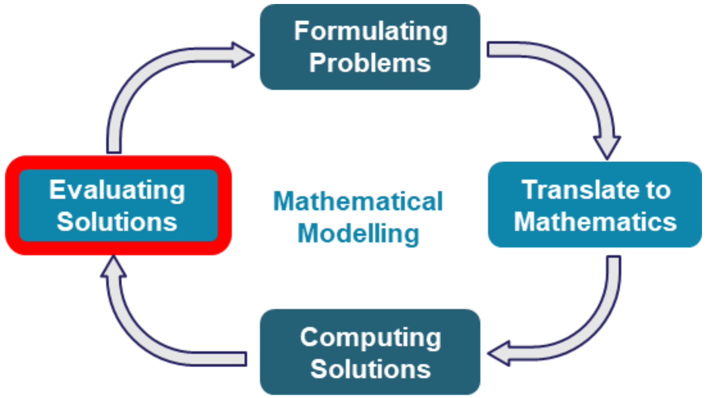


What are the limitations to this model?
How does it compare to real-life data?
How could we refine the model?

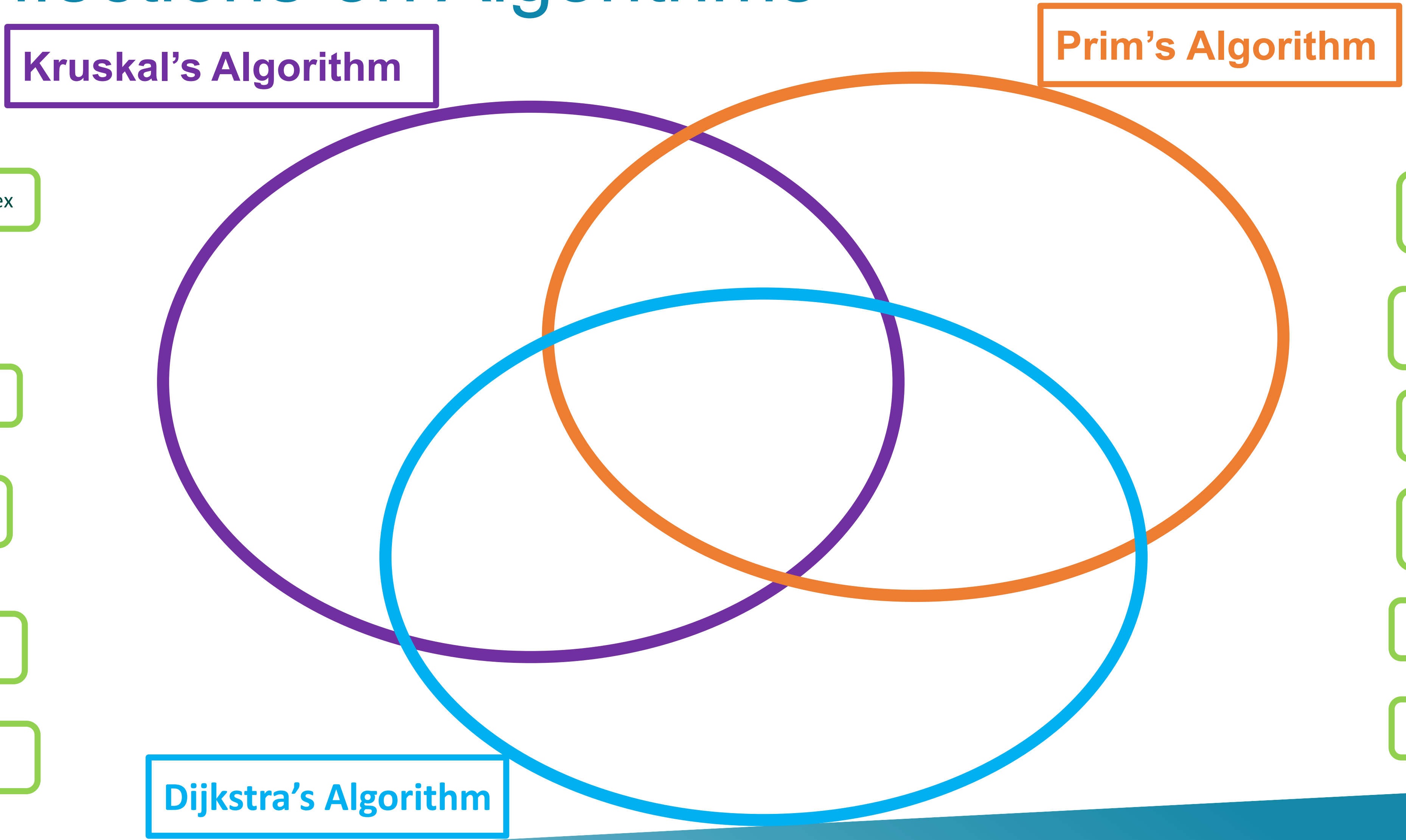
"refine a model and use it to predict a better solution to the problem; iterate the process " Specification p. 16

Heathrow
Paddington
Euston
King's Cross
Tott Hale
Stansted

100 mins



Reflections on Algorithms



- Starts with a vertex
- Creates a MST
- Starts with an edge
- More suitable for dense graphs
- Can be refined to improve the model
- Could be integrated with other strands

- Identifies a shortest path
- Follows a step-by-step method
- Follows a formal algorithm
- Can be used in Mathematical Modelling
- Used to solve authentic problems
- Other (Type your comment)

Reflection

What did you notice about the learning and teaching approaches in this session?

What are some of the benefits for students of using approaches like this?

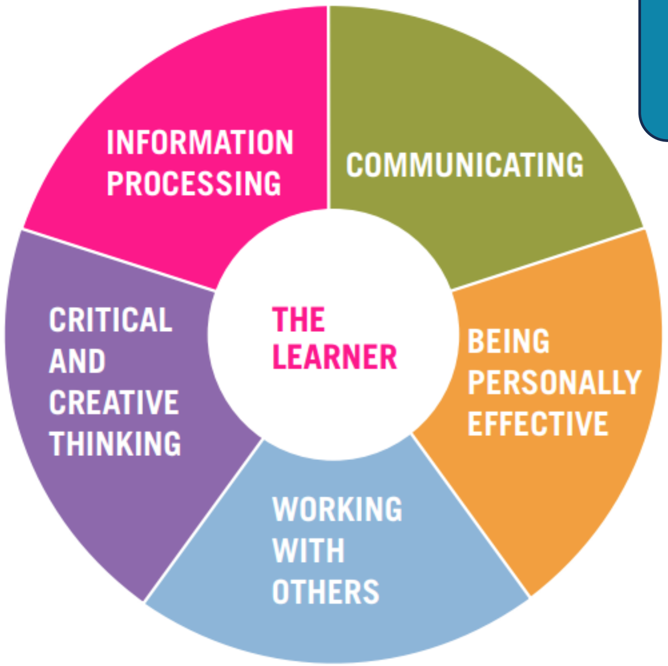


Figure 5: Key skills of senior cycle

Oide

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Supporting the Professional Learning of School Leaders and Teachers

Session 2

Mathematically Modelling Two-Particle Kinematics problems

By The End of This Session You Will Have:

Developed a deeper understanding of how to present problems which will develop students' modelling skills and competencies

Engaged in the four stages of the modelling cycle to develop students' understanding of two-particle problems

Experienced a constructivist teaching approach to actively involve students in deriving the equations for constant acceleration

Reflected on approaches to planning teaching and learning tasks

Interpreting a Real-World Problem

2

Concepts through Modelling

“Engaging with real problems is motivating for students; it allows them to see the relevance of mathematics to situations that are important in their lives.” Specification p. 13

Problem Statement:

James is a passenger on board a bus in London. He observes a man running to catch the bus as they are taking off from the bus stop.

He sees the man give up, but wonders, was it possible for the man to catch the bus?

Can the man catch the bus?

2

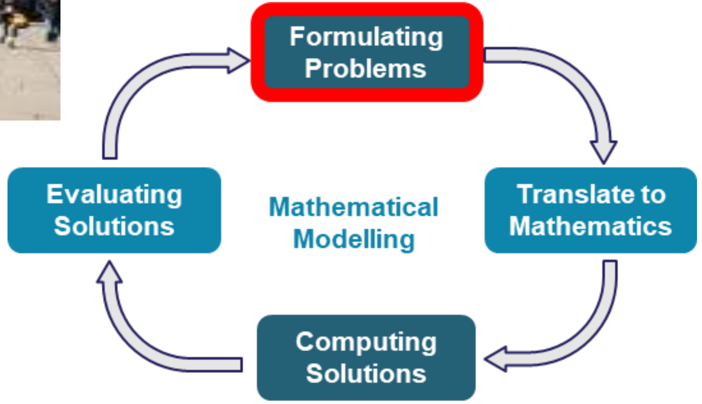
Concepts through Modelling

“Modelling problems require the solver to research the situation themselves, make reasonable assumptions, decide which variables will affect the solution, and develop a model that provides a solution that best describes the situation.”
Specification p. 10

Problem Statement:

James is a passenger on board a bus in London. He observes a man running to catch the bus as they are taking off from the bus stop.

He sees the man give up, but wonders, was it possible for the man to catch the bus?



What background information is required by students to model an answer to this question?

Analysing the motion of the bus

2

Concepts through Modelling

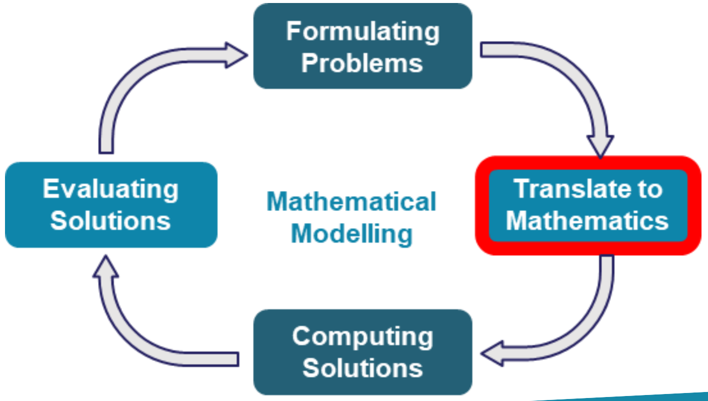
As the bus travels between the next two stops, James uses google maps data to track the motion of the bus.

Prior JC knowledge
What might students do next with this information, based on their JC Maths knowledge?
How might they draw the corresponding distance-time graph?
How might they describe the different stages of motion?

Time (s)	Distance (m)
0	0
20	80
25	132.5
60	587.5
85	750

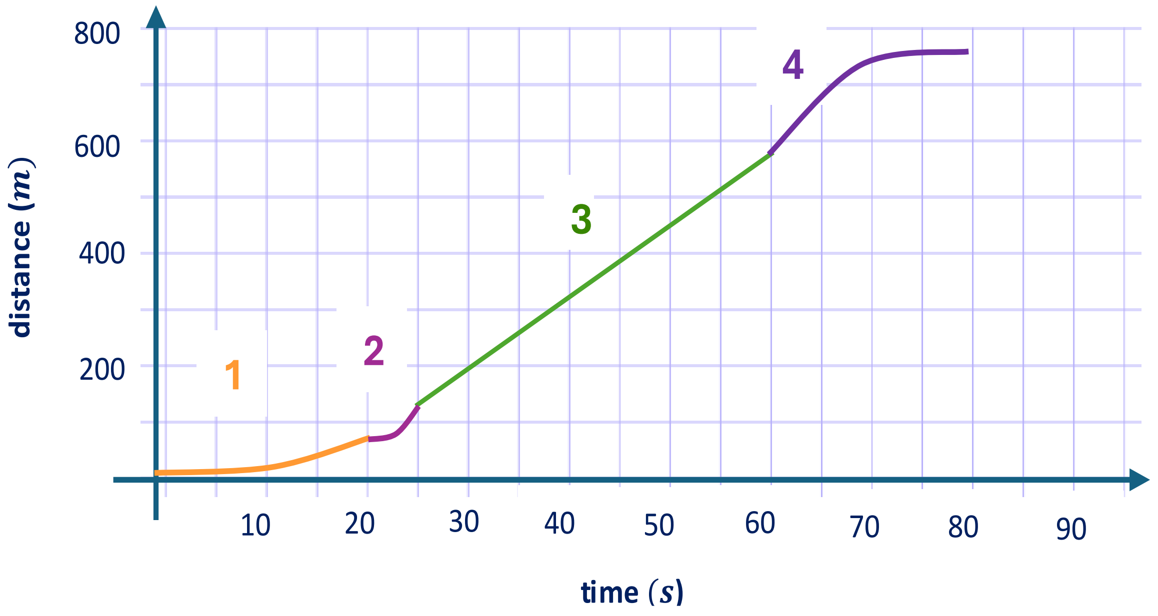
"use abstraction to describe systems and to explain the relationship between wholes and parts"
Specification p. 16

Google Maps



Building on prior knowledge

2 Concepts through Modelling



"describe the motion of a particle in 1D [In a straight line] using words, diagrams, numbers, graphs and equations" Specification p. 16

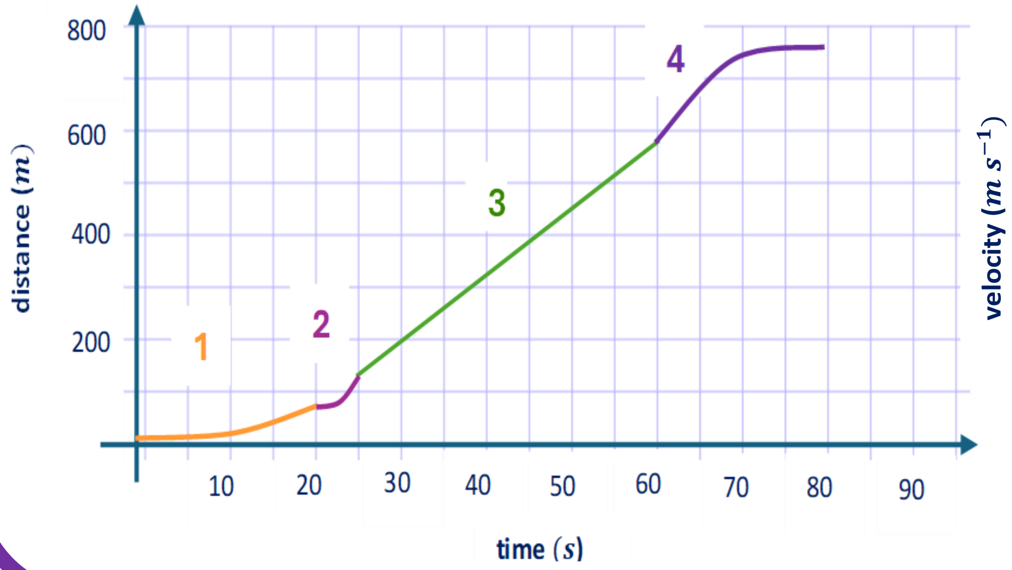
Time (s)	Distance (m)
0	0
20	80
25	132.5
60	587.5
85	750

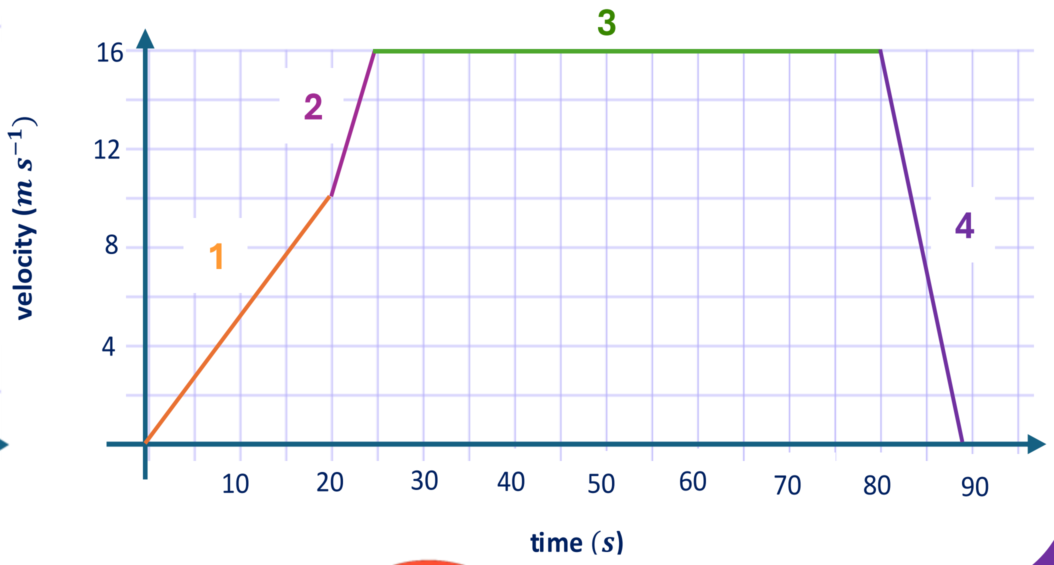
Building on prior knowledge

2

Concepts through Modelling

"Learning outcomes promote teaching and learning processes that develop students' knowledge and understanding incrementally"
Specification p. 14





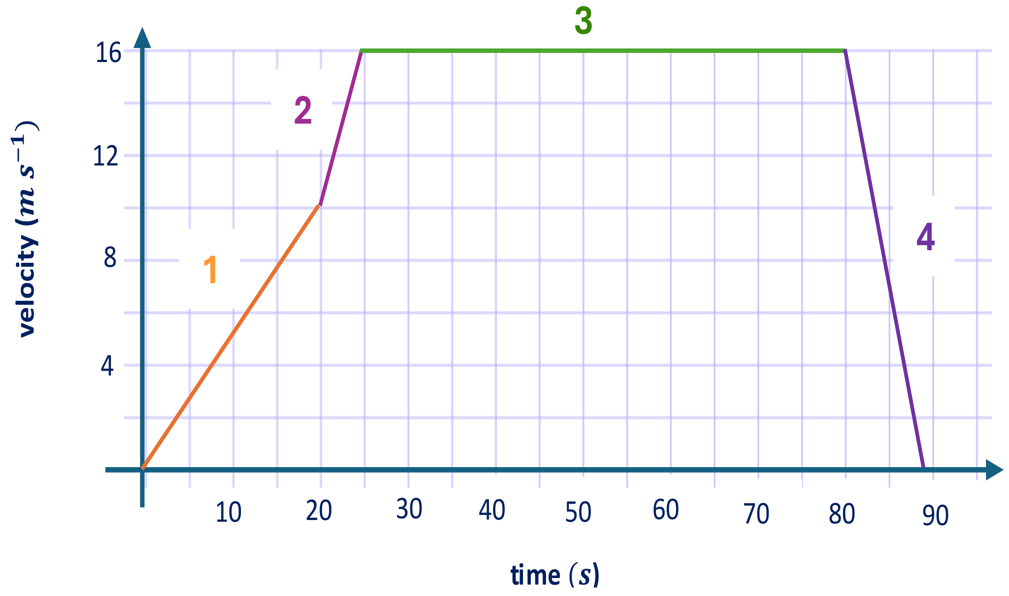
How do the graphs of the bus's motion relate to one another?

Building on prior knowledge

2

Concepts through Modelling

By interpreting the different sections of the bus's velocity-time graph we can learn a lot about it's motion



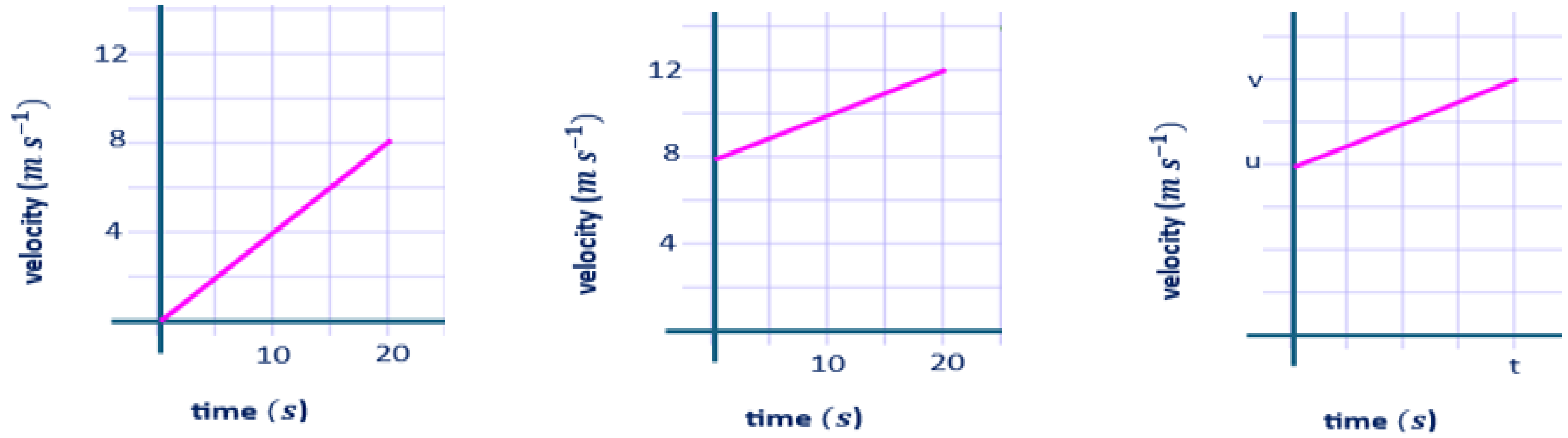
Section of the Graph	Slope	Velocity	Acceleration
1	Positive	Increasing	Positive
2	Positive	Increasing	Positive
3	Zero	Constant	Zero
4	Negative	Decreasing	Negative

Using the motion of the bus to create an expression for final velocity, v

2

Concepts through Modelling

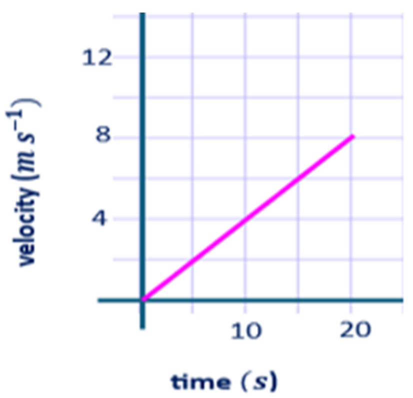
On the rest of the bus journey, James kept track of how long it took the bus to reach certain velocities . Can you create a general expression for final velocity, *v*, involving the parameters in the graph?

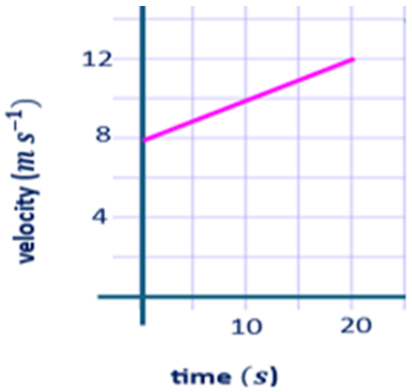


Using the motion of the bus to create an expression for final velocity, v

2

Concepts through Modelling





acceleration = $\frac{Change\ in\ velocity}{Change\ in\ time}$ = slope = $\frac{Rise}{Run}$

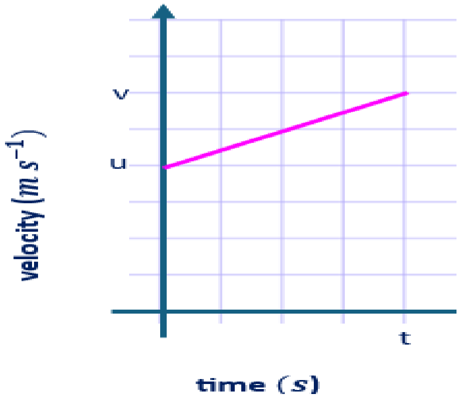
$$a = \frac{8\ - 0}{20 - 0}$$

$$a = \frac{12\ - 8}{20 - 0}$$

$$a = \frac{2}{5}\,m\,s^{-2}$$

$$a = \frac{1}{5}\,m\,s^{-2}$$

Generalising



$$a = \frac{v\ - u}{t - 0}$$

$$at = v\ - u$$

$$\boldsymbol{u + at = v}$$

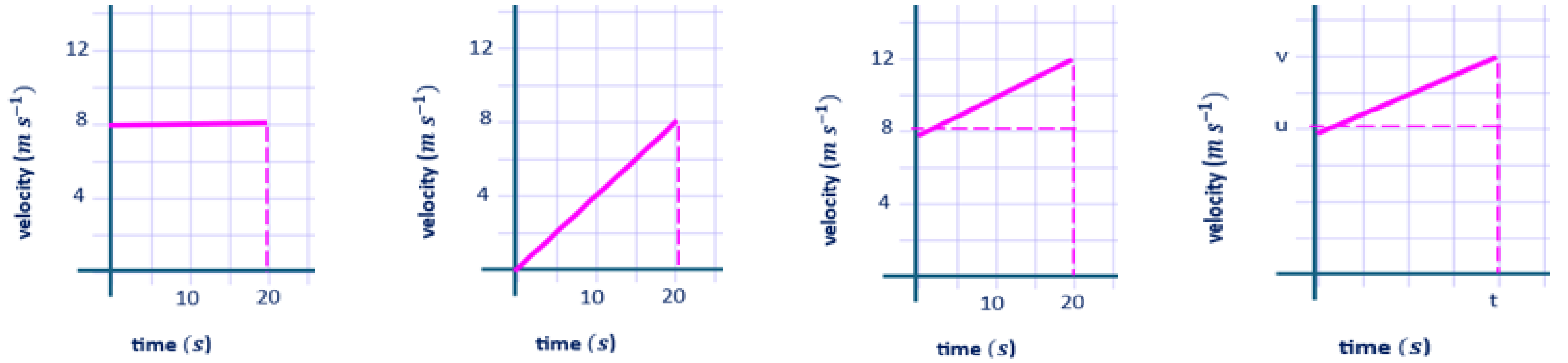
Connecting velocity-time graphs with distance travelled

2

Concepts through Modelling

"The focus on the experiential approach to teaching and learning, which is central to applied mathematics, means that students can be engaged in learning activities that complement their own needs and ways of learning."
Specification p. 14

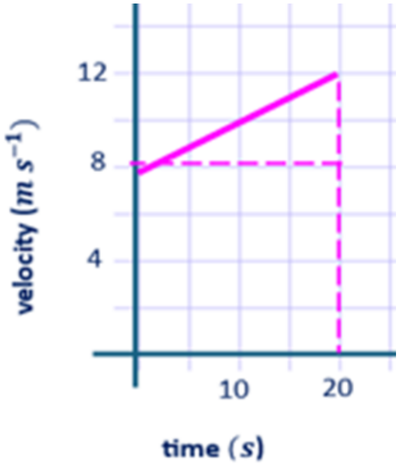
In the velocity-time graphs shown, what conclusions could you make about the distance travelled in each?



Using the motion of the bus to create an expression for displacement, s

2

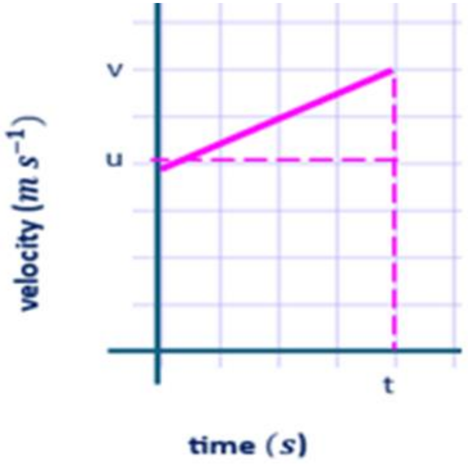
Concepts through Modelling



Area under graph from t = 0 s to t = 20 s

s_{20} = Area of ■ + Area of ◢
s_{20} = (20)(8) + ½ (20)(12 - 8)
s_{20} = 160 + 40
s_{20} = 200 m

Generalising



Area under graph from t = 0 s to t = t s

s_t = Area of ■ + Area of ◢
s_t = (u)(t) + ½ (t)(v - u)
s_t = ut + ½ (t)(at)
s_t = ut + ½ at^2

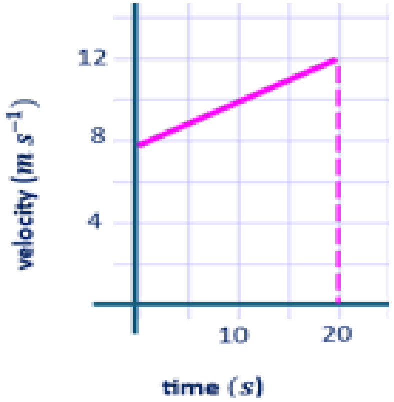
v = u + at

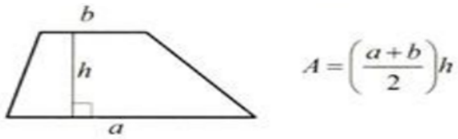
Oide

2

Using the motion of the bus to construct $v^2 = u^2 + 2as$

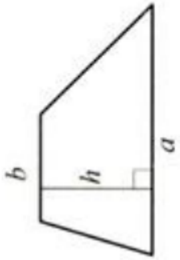
Concepts through Modelling



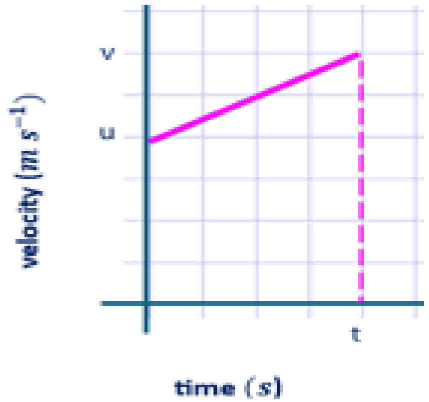


$s_{20} = (\frac{8+12}{2})(20)$

$s_{20} = 200\ m$



Generalising



$s_t = (\frac{u+v}{2})(t)$

v = u + at

also

$s_t = (\frac{u+v}{2})\left(\frac{v-u}{a}\right)$

$s_t = \frac{v^2 - u^2}{2a}$

$v^2 = u^2 + 2as$

Can the man catch the bus?

2

Concepts through Modelling

Formulating the problem:
- ❑ What variables (factors) are relevant to the problem?
- ❑ Can you simplify the problem to make it more manageable?
- ❑ What assumptions will you make?

How might your students model a solution to this question?

Translating to Mathematics:
- ❑ What mathematical approach will you use to solve the problem and why?
- ❑ Where will your assumptions and variables be used in your model?

Computing solution:
- ❑ How did you calculate your solution and what conclusions can you make from it?
- ❑ Explain the relationship between your solution and the original problem statement

Evaluate the Solution:
- ❑ Does your answer make sense?
- ❑ How accurate/reliable is your solution based on your assumptions?
- ❑ Can you refine your assumptions to improve your solution?

Can the man catch the bus?

Possible student approach

Formulating the problem:
- ❑ What variables (factors) are relevant to the problem?
- ❑ Can you simplify the problem to make it more manageable?
- ❑ What assumptions will you make?

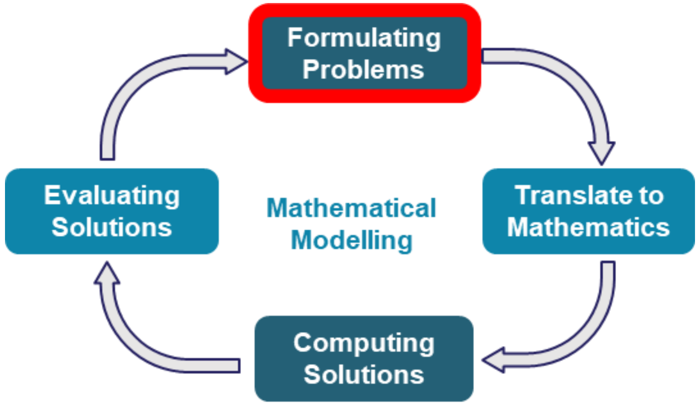
Refined Question:

What is the maximum distance the bus can be ahead of the man so he can catch it?

Assumptions:
- The man is running at a constant velocity
- Time to jump onto the bus is negligible
- The bus is accelerating at a constant rate from rest

Variables/Constants:
- Length of bus = 12 m (research)
- $v_{man} = 4\ m\ s^{-1}$ (research)
- $a_{bus} = 0.5\ m\ s^{-2}$ (average of all data collected)
- Distance the bus is ahead at start = d
- The time the man catches the bus = t

One of many approaches. Some students may choose to work with the man's velocity as the variable in question.



Can the man catch the bus?

Possible student approach

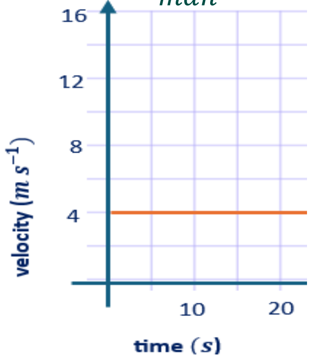
Translating to Mathematics:

- ❑ What mathematical approach will you use to solve the problem and why?
- ❑ Where will your assumptions and variables be used in your model?

d

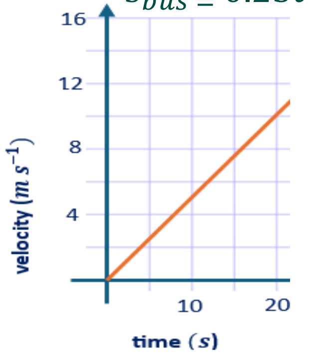
In order for the man to catch the bus he must run the distance they are apart and also the distance the bus has travelled in that time.

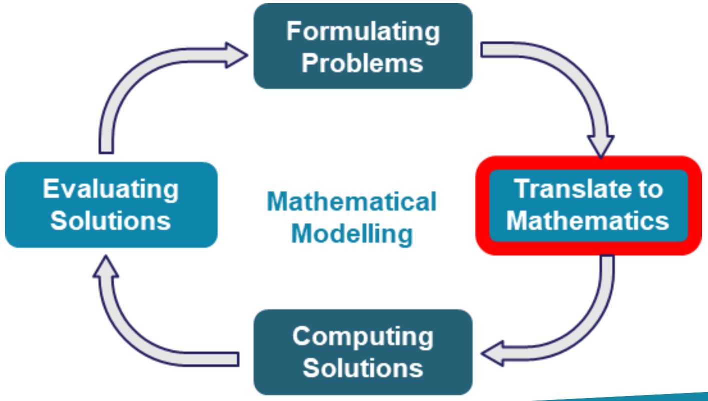
$s_{man} = 4\,t$



$s_{man} = s_{bus} + d$

$s_{bus} = 0.25t^2$





Can the man catch the bus?

Possible student approach

Computing solution:
- ❑ How did you calculate your solution and what conclusions can you make from it?
- ❑ Explain the relationship between your solution and the original problem statement

$s_{man} = s_{bus} + d$

1. Trial different gap distances:

@ d = 12 m (man is one full bus length behind)

$4t = 0.25t^2 + 12$

$0 = 0.25t^2 - 4t + 12$

$t = \frac{4 \pm \sqrt{(-4)^2 - 4(0.25)(12)}}{2(0.25)}$

t = 4 s and t = 12 s

Man will catch the bus after 4 seconds of running

@ d = 20 m (test slight increase in gap distance)

$4t = 0.25t^2 + 20$

$0 = 0.25t^2 - 4t + 20$

$t = \frac{4 \pm \sqrt{(-4)^2 - 4(0.25)(20)}}{2(0.25)}$

$t = \frac{4 \pm \sqrt{-4}}{2(0.25)}$ No solution Man won't catch the bus

2. Find greatest gap:

$t = \frac{4 \pm \sqrt{(-4)^2 - 4(0.25)(d)}}{2(0.25)}$

discriminant ≥ 0 for solutions

$\therefore (-4)^2 - 4(0.25)(d) \geq 0$

$16 \geq d$

∴ If the gap is 16 m or less, the man can catch the bus, running at a constant rate of $4\ m\ s^{-1}$.
If the gap is **16 m** the man will **"just" catch the bus,** based on assumptions.

3. Time it takes:

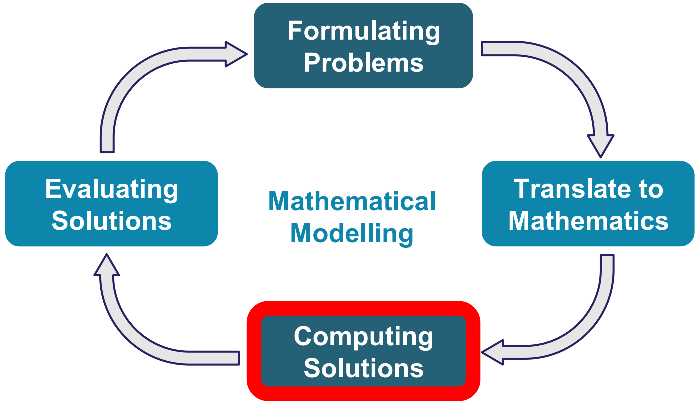
$t = \frac{4 \pm \sqrt{(-4)^2 - 4(0.25)(16)}}{2(0.25)}$

$t = \frac{4 \pm \sqrt{0}}{0.5}$ ∴ **t = 8s**

4. Velocity of bus:

$\therefore v_{man} = v_{bus}$ @ moment when man "just" catches bus

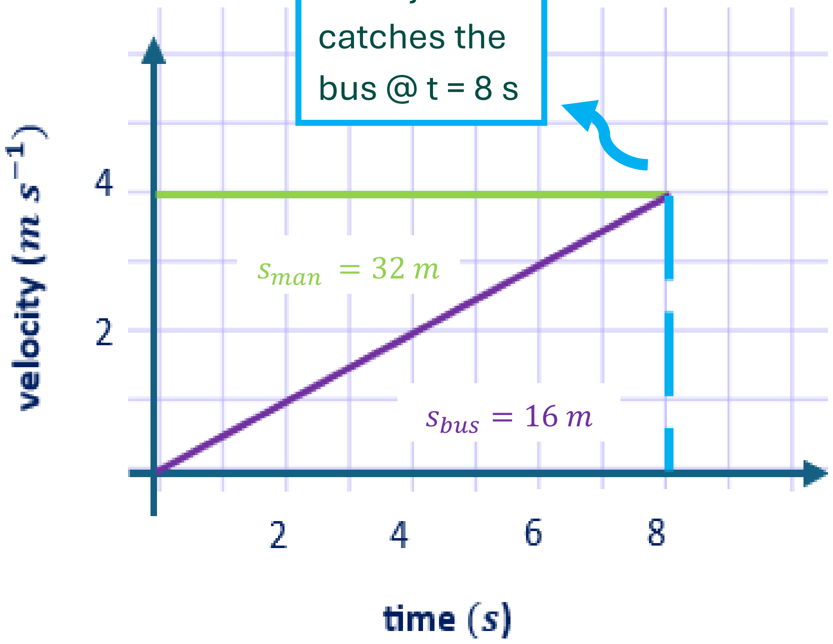
$v = \frac{1}{2}(8) = 4\ m\ s^{-1}$



Can the man catch the bus?

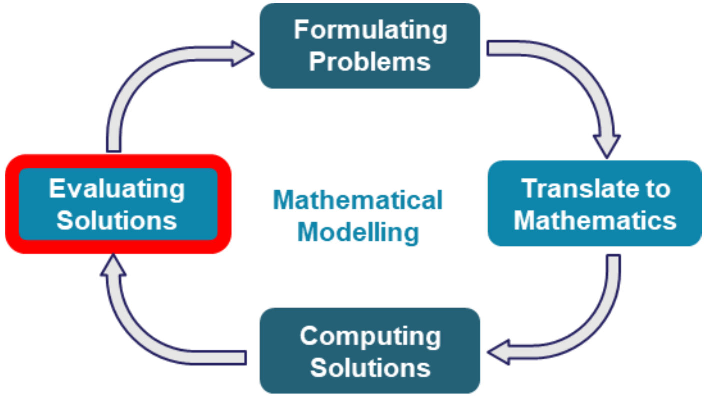
Possible student approach

Evaluate the Solution:
- ❑ Does your answer make sense?
- ❑ How accurate/reliable is your solution based on your assumptions?
- ❑ Can you refine your assumptions to improve your solution?



Validity of solution:
- This solution makes sense in the context of the assumptions made

Refining Solution:
- In reality, it is unlikely the man will run at a constant velocity – could account for varying velocity in next iteration
- The motion of jumping on the bus could be taken into account to make the model more realistic
- The time it takes seems very low – may need to increase the acceleration of the bus



Reflection

What did you notice about the learning and teaching approaches in this session?

What are some of the benefits for students of using approaches like this?

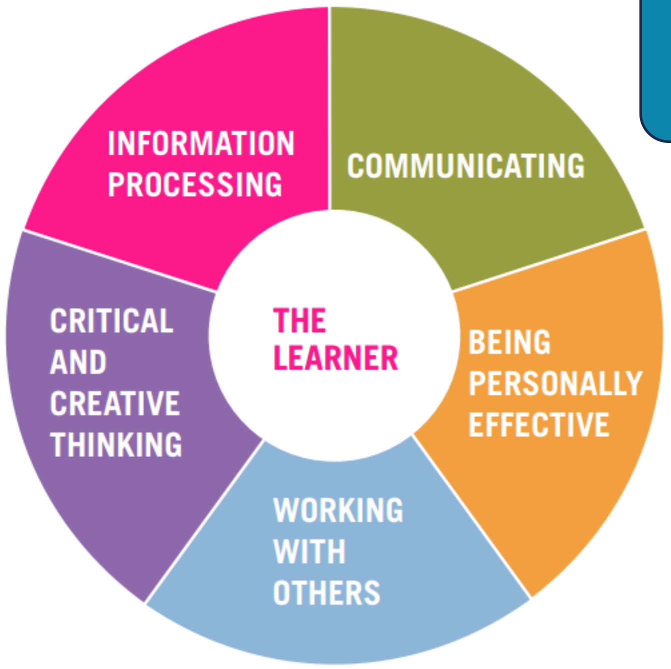


Figure 5: Key skills of senior cycle

Oide

Tacú leis an bhFoghlaim Ghairmiúil i measc Ceannairí Scoile agus Múinteoirí

Supporting the Professional Learning of School Leaders and Teachers

Session 3

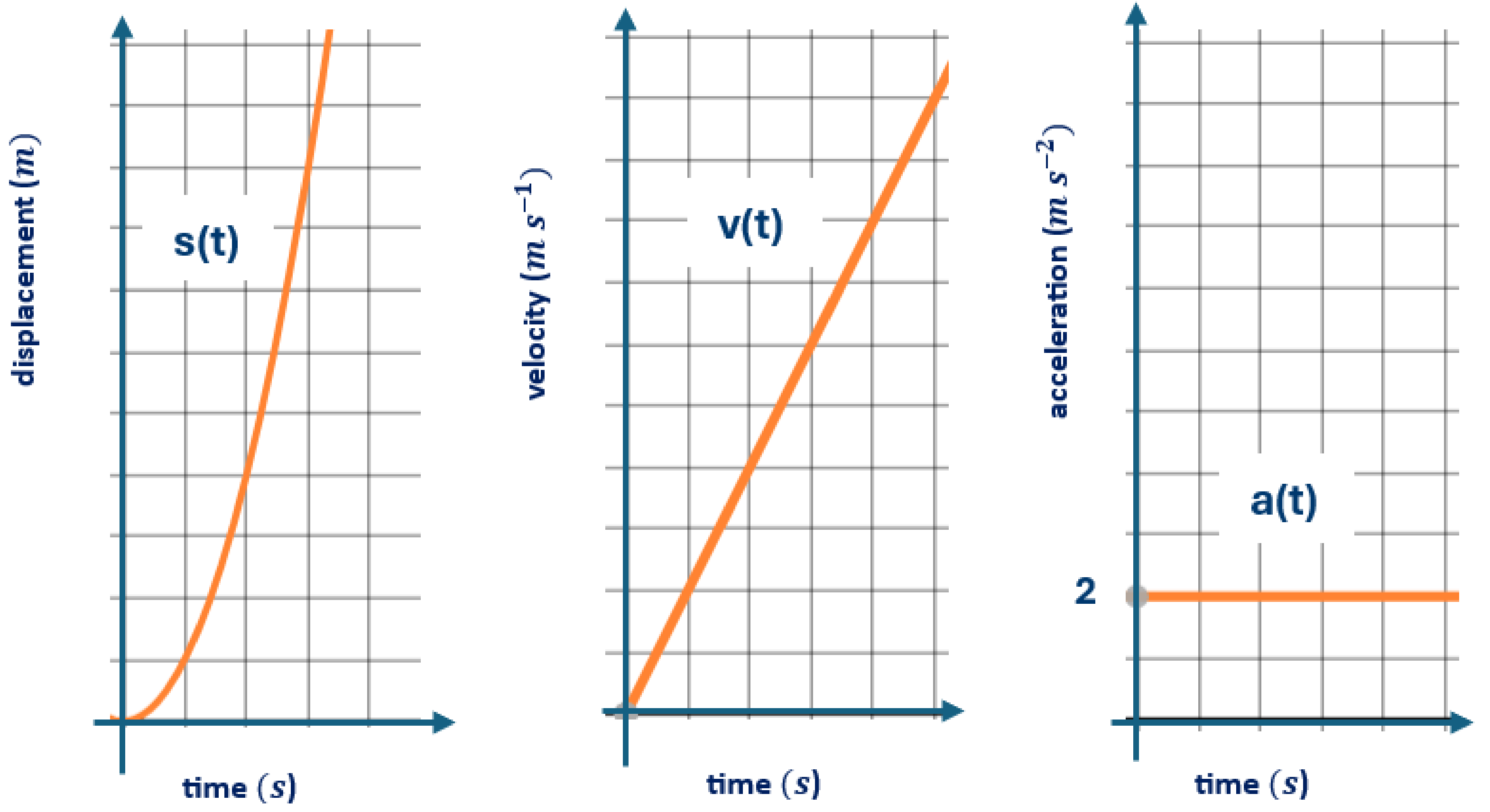
Exploring Differential Equations through the lens of Mathematical Modelling

By The End of This Session You Will Have:

- Engaged in approaches to building on students' prior knowledge and making connections between strands
- Experienced a constructivist teaching approach to actively involve students in deriving the kinematics equations using calculus
- An understanding of how Differential Equations can be developed and formalised through authentic modelling problems
- Engaged in the four stages of the modelling cycle to develop students' understanding of differential equations

Building on prior knowledge

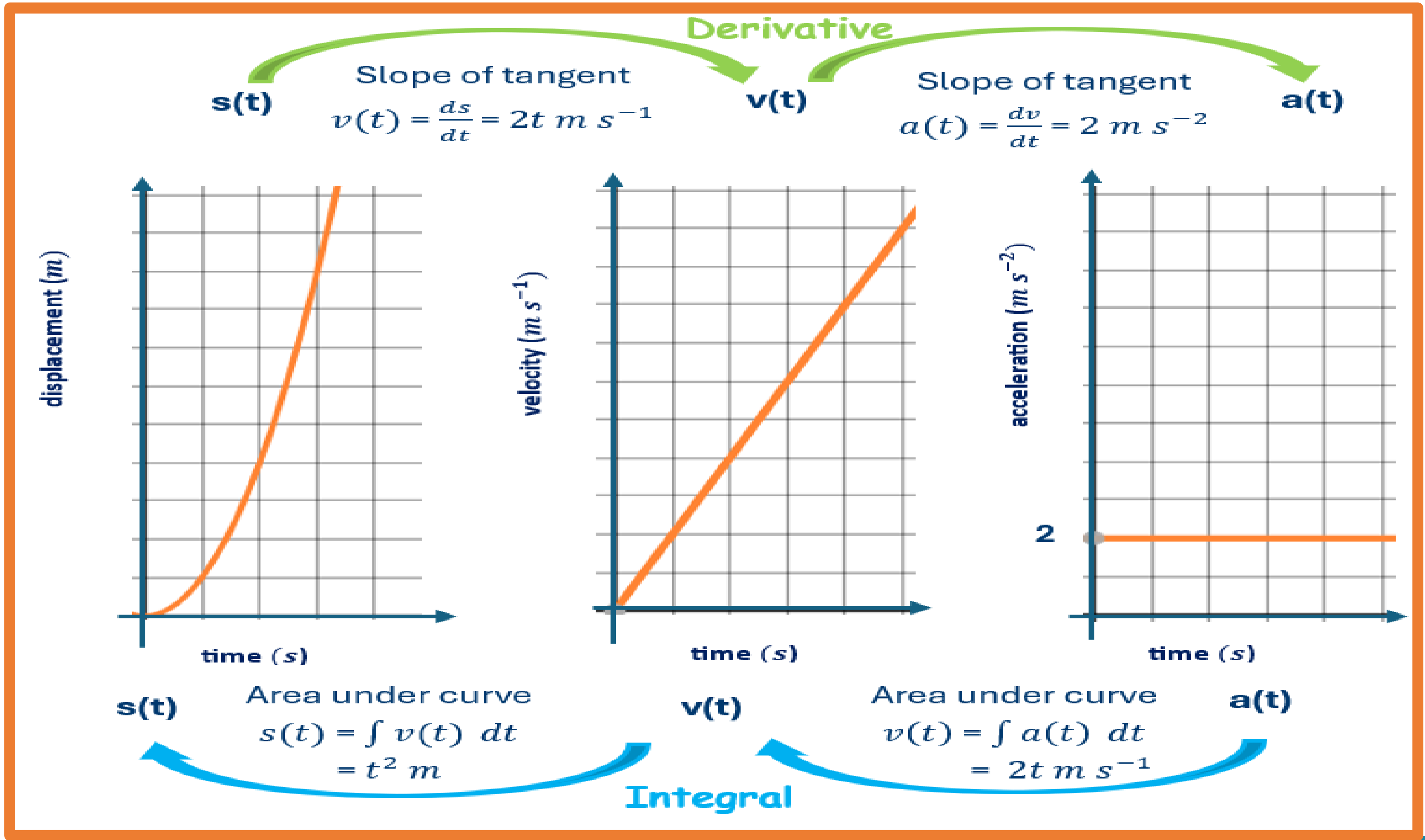
The graphs below all describe the same stage of motion.



How might your students approach finding the functions s(t), v(t) and a(t), in terms of t?

How could we develop that knowledge further, to introduce the use of calculus in kinematics?

Building on prior knowledge



Guided Discovery- Deriving the kinematics formulae using calculus

$$\boldsymbol{a = \frac{dv}{dt}}$$

$$\int a\,dt = \int 1\,dv$$

$$at + c = v$$

How will we determine the unknown constant of integration?

We require information about the motion.

Using the condition for initial velocity: @ $t = 0, v = u$.

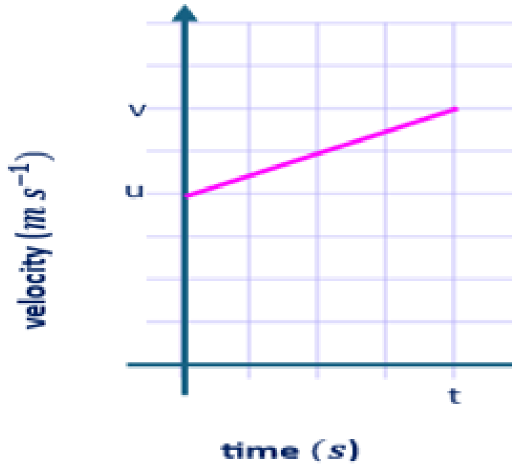
$$a(0) + c = u$$

$$c = u$$

Substituting: $at + u = v$

$$\therefore \boldsymbol{v = u + at}$$

"..derive the kinematics formulae of motion using calculus"
Specification p. 18



Guided Discovery- Deriving the kinematics formulae using calculus

v = $u + at$

$$v = \frac{ds}{dt}$$

$$u + at = \frac{ds}{dt}$$

$$(u + at)dt = ds$$

$$\int (u + at)dt = \int 1\, ds$$

$$ut + \frac{at^2}{2} + c = s$$

Again, we require information about the motion to determine the value of the constant of integration.

Using the condition for measuring displacement from a starting position:

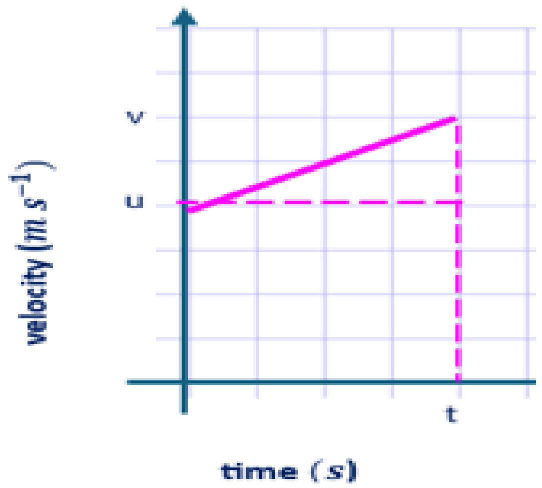
$$@\, t = 0, s = 0.$$

$$u(0) + \frac{a(0)^2}{2} + c = 0$$

$$c = 0$$

$$\therefore\ \boldsymbol{s = ut + \frac{1}{2}at^2}$$

“..derive the kinematics formulae of motion using calculus”
Specification p. 18



Guided Discovery- Deriving the kinematics formulae using calculus

Oide

What about the equation $\boldsymbol{v^2 = u^2 + 2as}$?

Firstly, we need a derivative for acceleration that relates velocity and displacement.

How might we do this using the derivatives of motion we know so far?

$$\boldsymbol{a = \frac{dv}{dt}} \text{ and } \boldsymbol{v = \frac{ds}{dt}}$$

Starting off with acceleration:

$$a = \frac{dv}{dt}$$

Links to Related Rates of Change- LCHL Maths

$$a\,.\,\boldsymbol{\frac{dt}{ds}} = \frac{dv}{dt}\,.\,\boldsymbol{\frac{dt}{ds}}$$

$$a\,.\,\frac{dt}{ds} = \frac{dv}{ds}$$

"..derive the kinematics formulae of motion using calculus"
Specification p. 18

$$a = \frac{dv}{ds}\,.\,\frac{ds}{dt}$$

$$a = \frac{dv}{ds}\,.\,v$$

$$\frac{ds}{dt} = v$$

$$\therefore \boldsymbol{a = v\frac{dv}{ds}}$$

Guided Discovery- Deriving the kinematics formulae using calculus

Oide

Using our derived derivative:

$$\boldsymbol{a = v\frac{dv}{ds}}$$

$$a\,ds = v\,dv$$

$$\int a\,ds = \int v\,dv$$

$$as + c = \frac{v^2}{2}$$

Again, we require information about the motion to determine the value of the constant of integration.

"..derive the kinematics formulae of motion using calculus"
Specification p. 18

Using the condition for calculating velocity from a starting position:

$$@\, s = 0, v = u.$$

$$a(0) + c = \frac{u^2}{2}$$

$$as + \frac{u^2}{2} = \frac{v^2}{2}$$

$$2as + u^2 = v^2$$

$$\boldsymbol{\therefore v^2 = u^2 + 2as}$$

Approaches To Mathematical Modelling in The Classroom

Oide

Recall

1

Concepts then Modelling

Explore a number of mathematical concepts through suitable tasks, word problems etc., then solve a rich modelling problem. In exploring these tasks, modelling competencies may also be developed.

Complete a full modelling cycle

Focus on a subset of competencies

2

Concepts through Modelling

Explore a rich modelling problem and, as the need arises, develop understanding of new mathematical concepts through instruction, guided discovery, research, etc.

Complete a full modelling cycle

Focus on a subset of competencies

Modelling the velocity of a runner

2

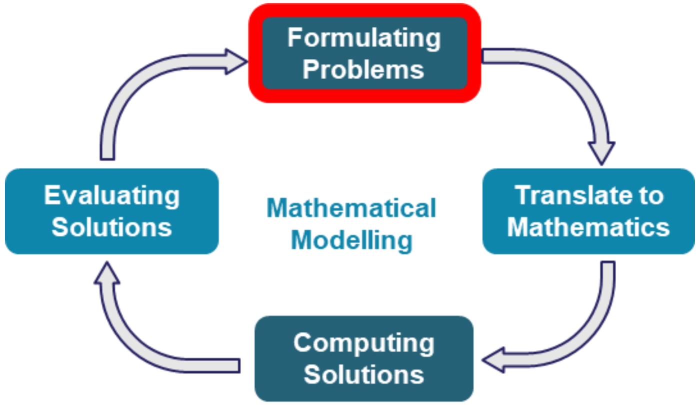
Concepts through Modelling

What variables/factors might your students consider?

How might they approach simplifying the problem?

How could this modelling problem build on prior knowledge of other Applied Mathematics topics?

"The focus on the experiential approach to teaching and learning, which is central to applied mathematics, means that students can be engaged in learning activities that complement their own needs and ways of learning."
Specification p. 14



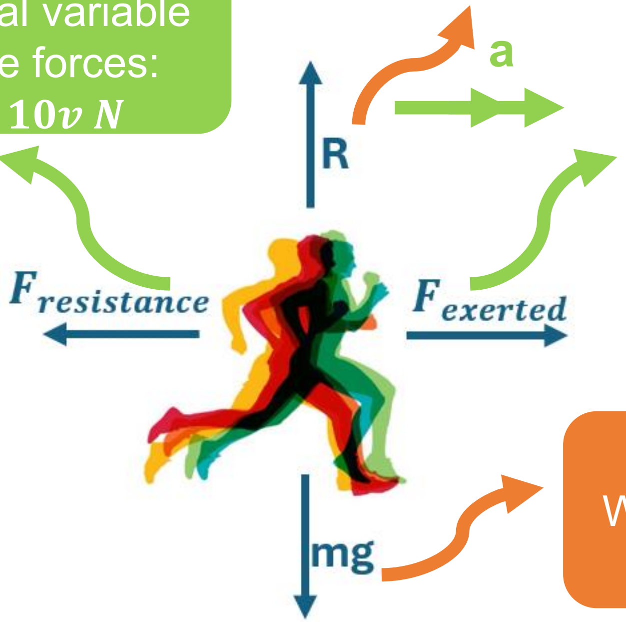
Modelling the velocity of a runner

2

Concepts through Modelling

Normal Reaction Force

Combined internal and external variable resistance forces: approx. $10v\ N$

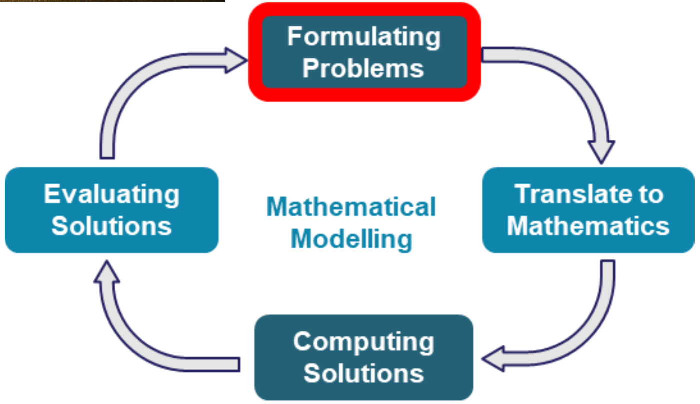


Total force exerted in the direction of motion: approx. $90\ N$

Weight Force: $80g\ N$

"draw free-body force diagrams for a particle on a smooth rigid fixed horizontal or inclined plane"

Specification p. 19



Modelling the velocity of a runner

2

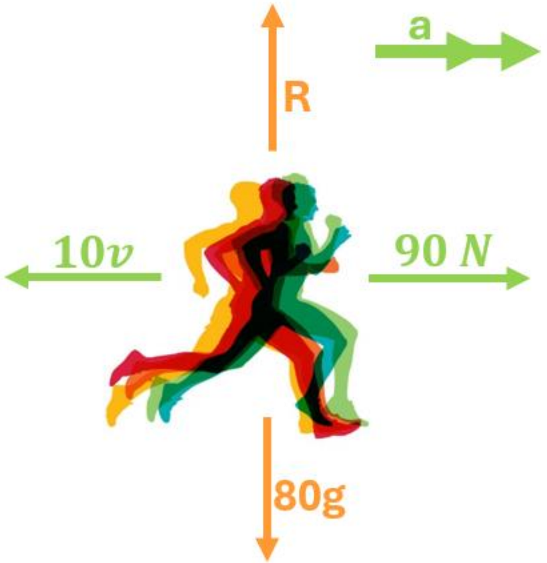
Concepts through Modelling

Forming an equation for acceleration:

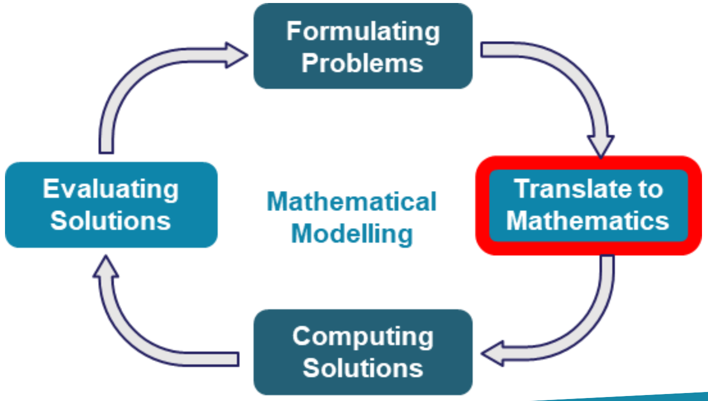
$$F_{net=ma}$$

$$90 - 10v = 80a$$

$$(1.125 - 0.125v)\ m\ s^{-2} = a$$



"solve dynamic problems involving resistive forces that are proportional to v^n n ∈ R"
Specification p.19



Modelling the velocity of a runner

2

Concepts through Modelling

"derive and interpret in context differential equations for real-world phenomena involving continuous change" *Specification p. 21*

Deriving and solving the differential equation:

$$a = \frac{dv}{dt}$$

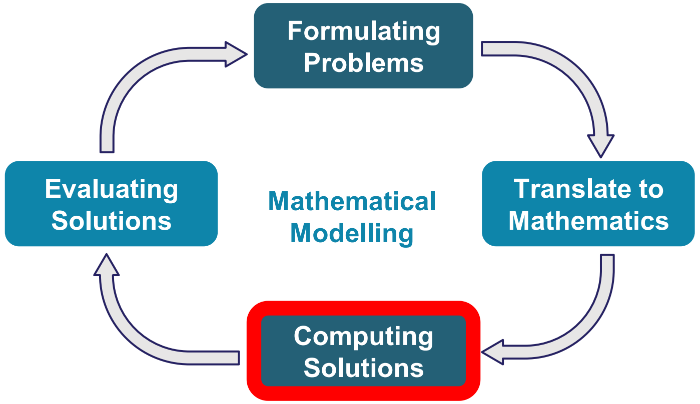
$$\frac{dv}{dt} = 1.125 - 0.125v$$

$$\frac{1}{1.125 - 0.125v}\,dv = 1\,dt$$

$$\int \frac{1}{1.125 - 0.125v}\,dv = \int 1\,dt$$

$@t = 0, v = 0$ (assumed condition)

$$\int_0^v \frac{1}{1.125 - 0.125v}\,dv = \int_0^t 1\,dt$$



Modelling the velocity of a runner

2

Concepts through Modelling

$$\int_0^v \frac{1}{1.125 - 0.125v}\,dv = \int_0^t 1\,dt$$

$$\left[\frac{\ln(1.125 - 0.125v)}{-0.125}\right]_0^v = [t]_0^t$$

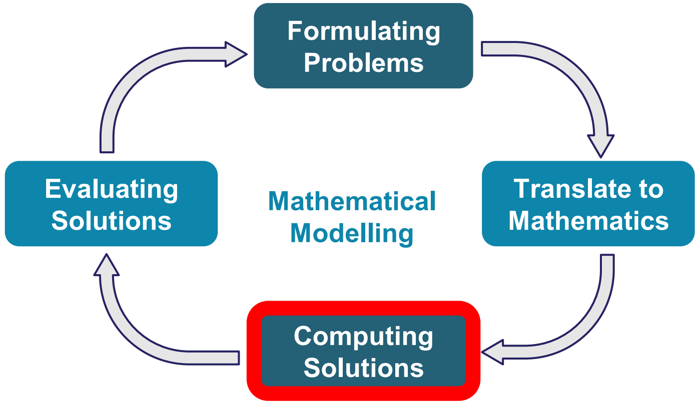
$$[-8\ln(1.125 - 0.125v)]_0^v = [t]_0^t$$

$$-8\ln(1.125 - 0.125v) + 8\ln(1.125 - 0.125(0)) = t - 0$$

$$-\ln(1.125 - 0.125v) + \ln(1.125) = \frac{t}{8}$$

$$\ln\left(\frac{1.125}{1.125 - 0.125v}\right) = 0.125t$$

"derive and interpret in context differential equations for real-world phenomena involving continuous change" *Specification p. 21*



Modelling the velocity of a runner

2

Concepts through Modelling

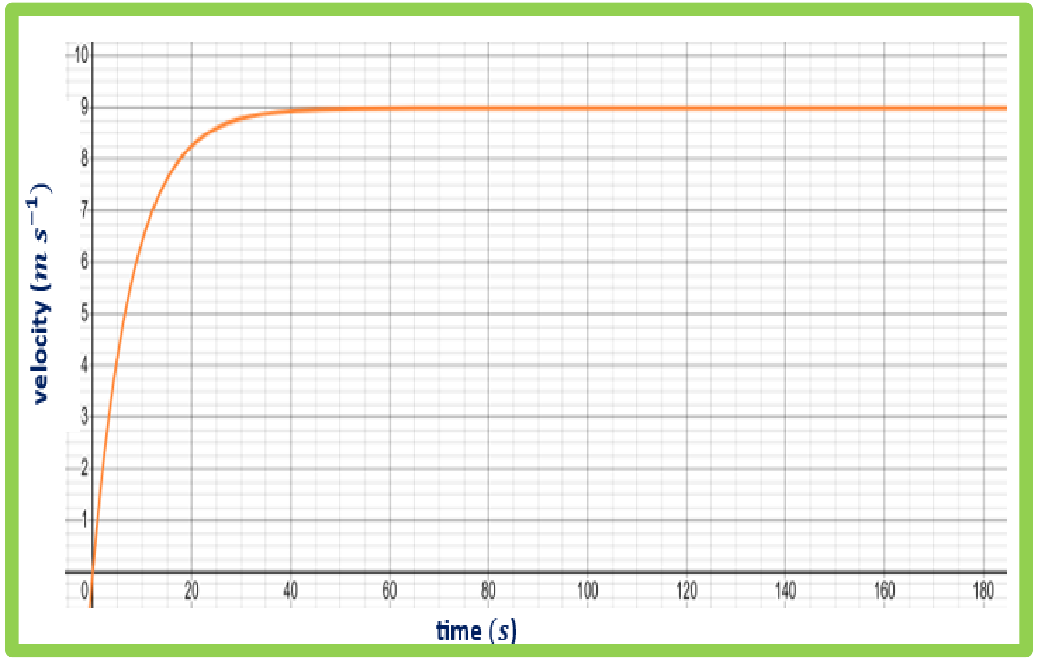
Using Desmos graphing calculator

$$\frac{1.125}{1.125 - 0.125v} = e^{0.125t}$$

$$\frac{1.125}{e^{0.125t}} = 1.125 - 0.125v$$

$$0.125v = 1.125 - 1.125e^{-0.125t}$$

$$\therefore v = 9 - 9e^{-0.125t}$$



"derive and interpret in context differential equations for real-world phenomena involving continuous change" *Specification p. 21*

Formulating Problems

Translate to Mathematics

Computing Solutions

Evaluating Solutions

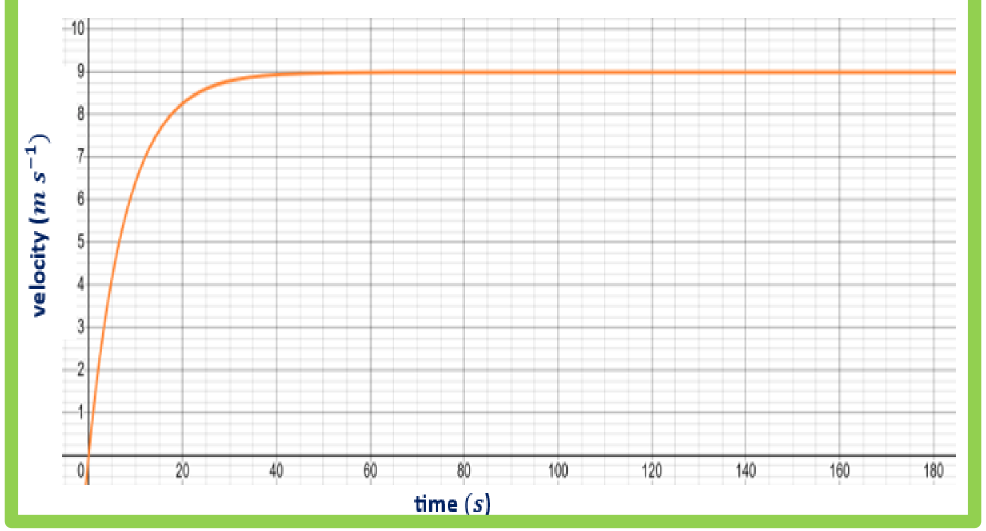
Mathematical Modelling

Modelling the velocity of a runner

2

Concepts through Modelling

$$v = 9 - 9e^{-0.125t} \quad m\,s^{-1}$$

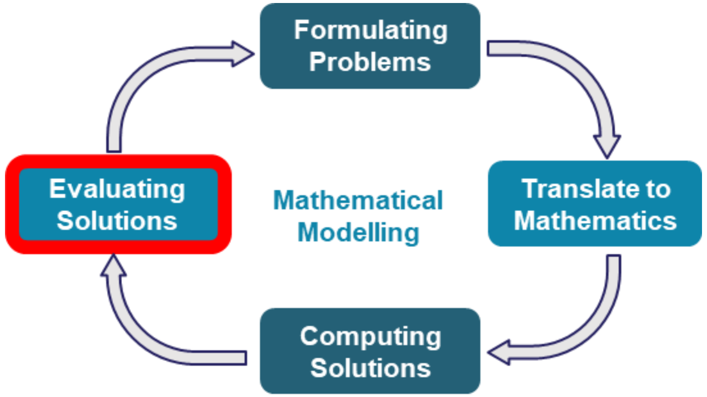


How might your students evaluate this result?

What real life comparisons could they make?

How could we utilise this problem to extend their learning?

"derive and interpret in context differential equations for real-world phenomena involving continuous change" *Specification p. 21*



Oide

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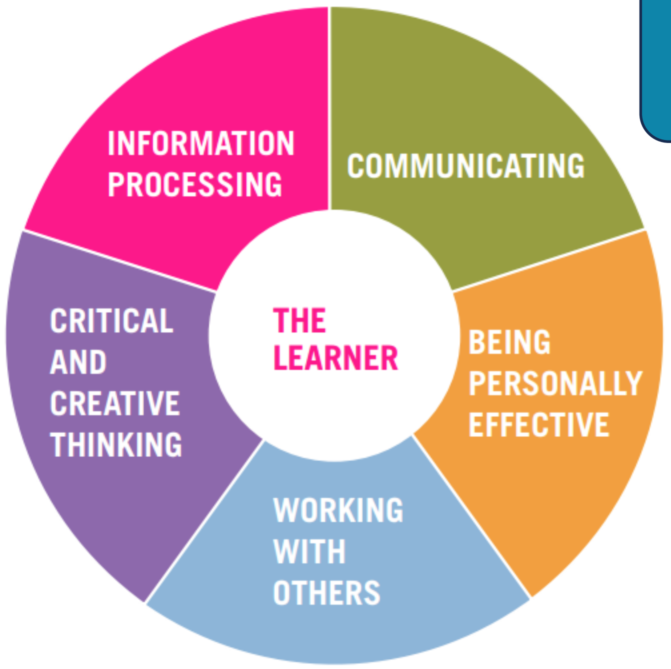

Figure 5: Key skills of senior cycle