Oide

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Supporting the Professional Learning of School Leaders and Teachers

# Building connected and integrated mathematical understanding in Geometry

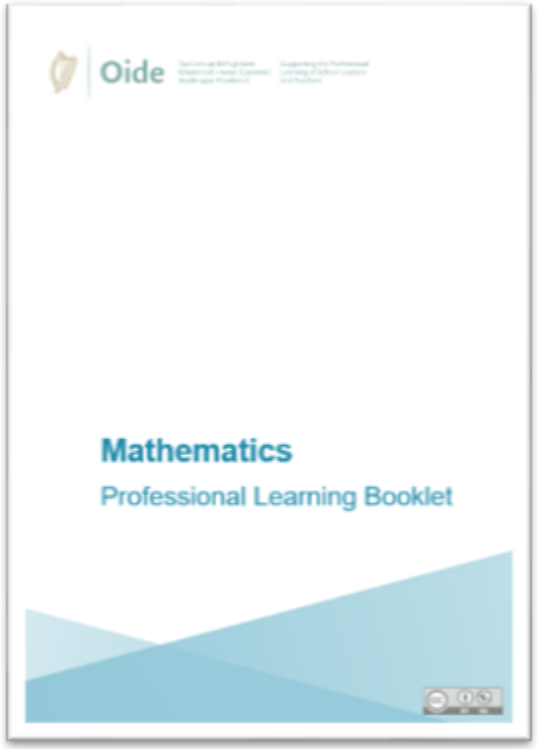
## Mathematics Professional Learning Event (PLE)

## 2024 - 2025

# Schedule

| | |
|---|---|
| 09:00 – 09:15 | Registration |
| **09:15 – 11:00** | **Session 1** - Developing students' geometric thinking |
| 11:00 – 11:30 | Break |
| **11:30 – 13:00** | **Session 2** – Student exploration of geometric concepts |
| 13:00 – 14:00 | Lunch |
| **14:00 – 15:45** | **Session 3** - Designing learning experiences |

# Resources




Digital Booklet

https://padlet.com/oideppmaths/oidemaths24

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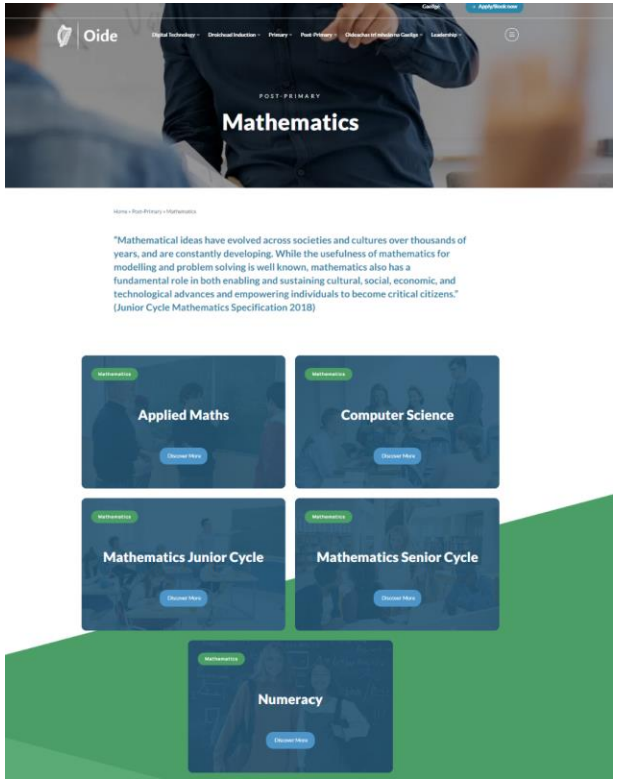
Supporting the Professional Learning of School Leaders and Teachers

# Introduction

# Mathematics & Computer Science

https://oide.ie/post-primary/home/mathematics/

- Applied Mathematics @OideAppliedMath
- Computer Science @Oide_CompSci
- Mathematics
- Numeracy




Mailing List: https://tinyurl.com/oidemaillist

# Mathematics & Numeracy

postprimarymaths@oide.ie

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Administrators:

Grainne Haughney

Rachel Whearity

# Key Message

Students need exposure to active learning to develop geometric thinking.

# Learning Intentions

- To consider how to develop students' geometric thinking

- To increase opportunities for meaningful student engagement with geometrical concepts

# Discussion 1

What challenges arise for teachers when teaching Geometry?

What are the barriers to students' learning of Geometry?

**10 mins**

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Supporting the Professional Learning of School Leaders and Teachers

# Session 1

## Developing students' geometric thinking

**Learning Intention:**

To consider how to develop students' geometric thinking.

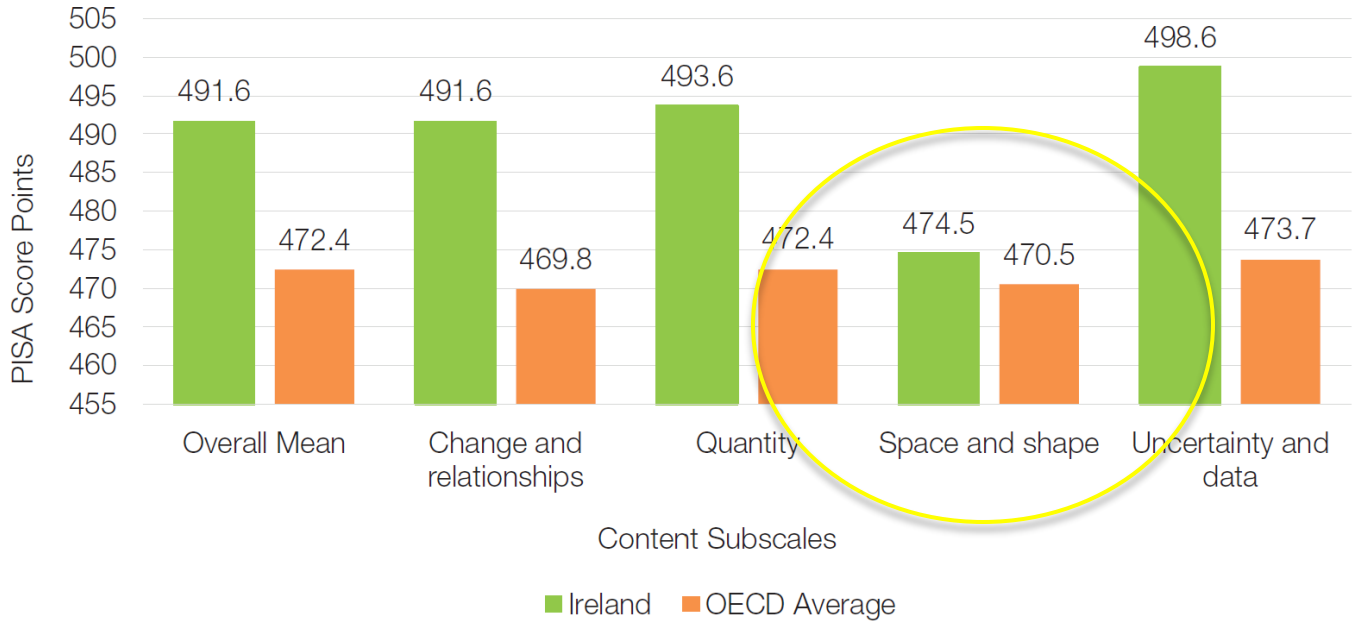
**Success Criteria:**

I have

- reflected on challenges in the learning and teaching of geometry
- been provided with an overview of pedagogical theory
- considered how this theory applies to my classroom practice
- engaged with geometric tasks and considered how these tasks provide access for diverse learners
- explored the role of mathematical language

# Overview of Key PISA/TIMMS Findings

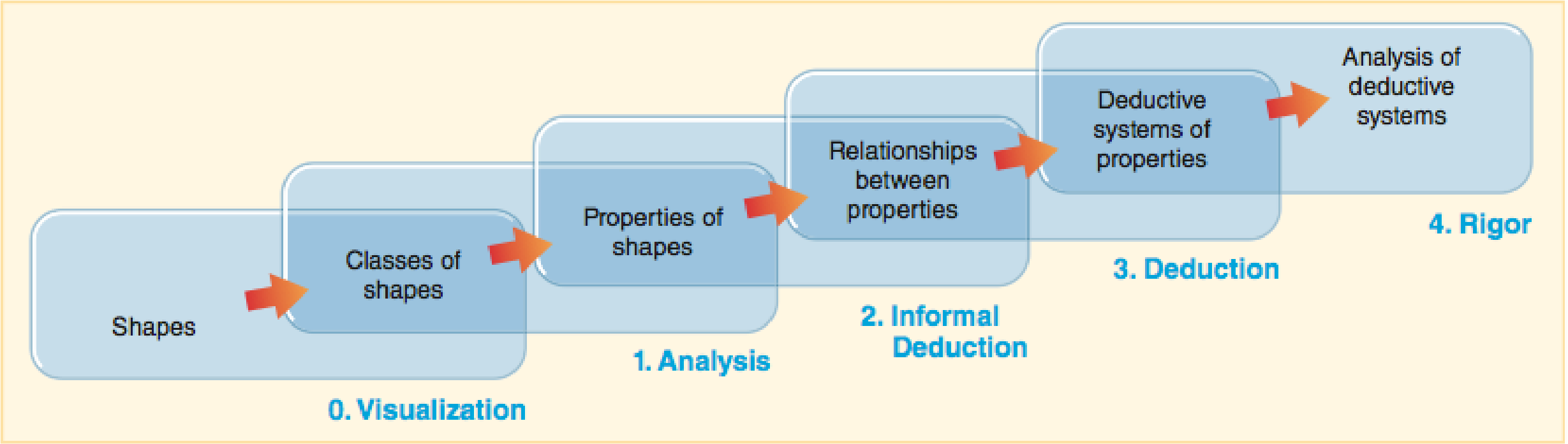

**Figure 3.5. Mean scores on the overall mathematics scale and the content subscales, in Ireland, and on average across OECD countries**




Source: OECD 2023c, Tables I.B1.2.1, I.B1.2.8, I.B1.2.9, I.B1.2.10 and I.B1.2.11.
Note: OECD average includes 37 OECD countries participating in PISA 2022.

ERC (2023) page 47

# Van Hiele Theory of Geometric Thought

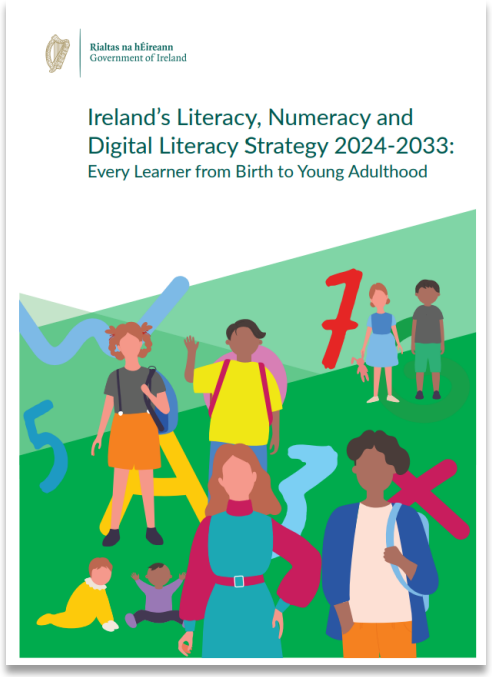



Van de Walle et al. (2020) pg 549

# Van Hiele Theory of Geometric Thought

| | Students at this level | |
|---|---|---|
| **Level 0 (Visualisation)** | • can distinguish individual shapes<br>• can not identify properties or classes | 'It looks like a square'<br>'A door is a rectangle'. |
| **Level 1 (Analysis)** | • Recognise that shapes have different properties<br>• consider shapes within a class | Identify rectangles regardless of size and orientation.<br>Work with properties of rectangles |
| **Level 2 (Informal Deduction)** | • see relationships between properties | If it is a square, it must be a rectangle.<br>A rectangle is a parallelogram with right angles |
| **Level 3 (Deduction)** | • Analyse informal arguments | Appreciate the need for proof |

# Literacy, Numeracy and Digital Literacy Strategy (2024)



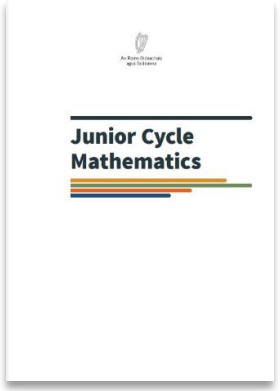

"*Numeracy is the ability to **observe, explore and manipulate** objects, and hear mathematical language. It involves using mathematical understanding and skills in a variety of contexts to describe, predict and explain phenomena, recognising the role that mathematics plays in the world*".

**Pillar 4:** Improving the learner experience through curriculum, pedagogy and assessment

**Pillar 5:** Supporting diverse learners to achieve their potential

DES (2024) page3 28, 42, 43

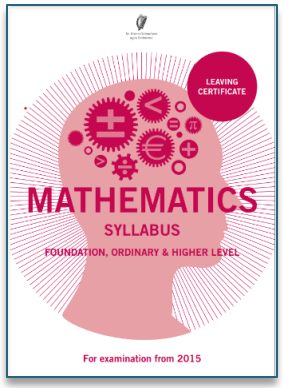
# Overview of Geometry




This strand focuses on **analysing** characteristics and properties of shapes

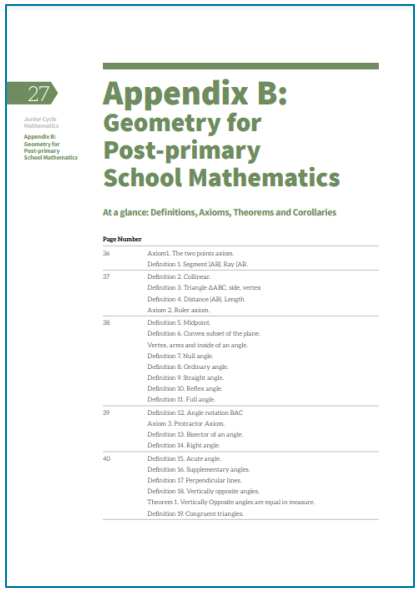
Learners

- use geometry to **model** and solve problems
- They develop mathematical arguments
- **explore** the concept of formal proof



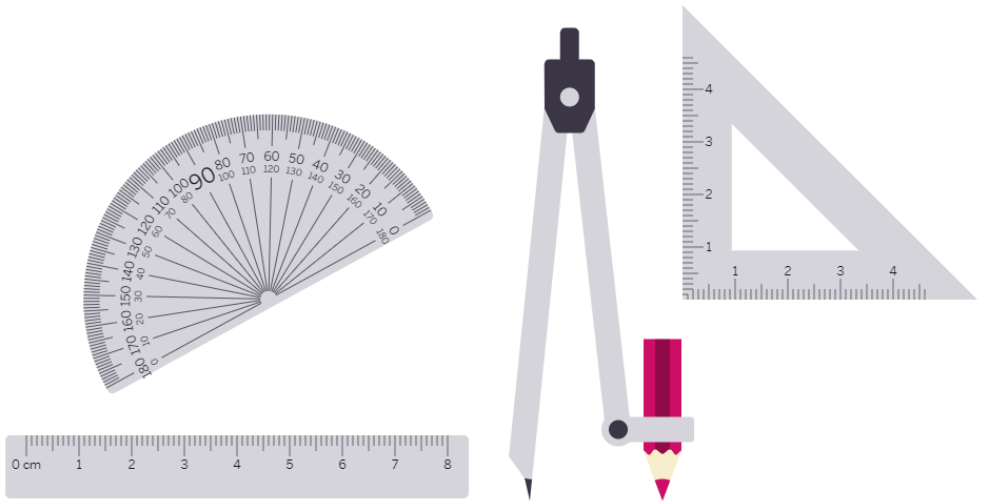

Learners should

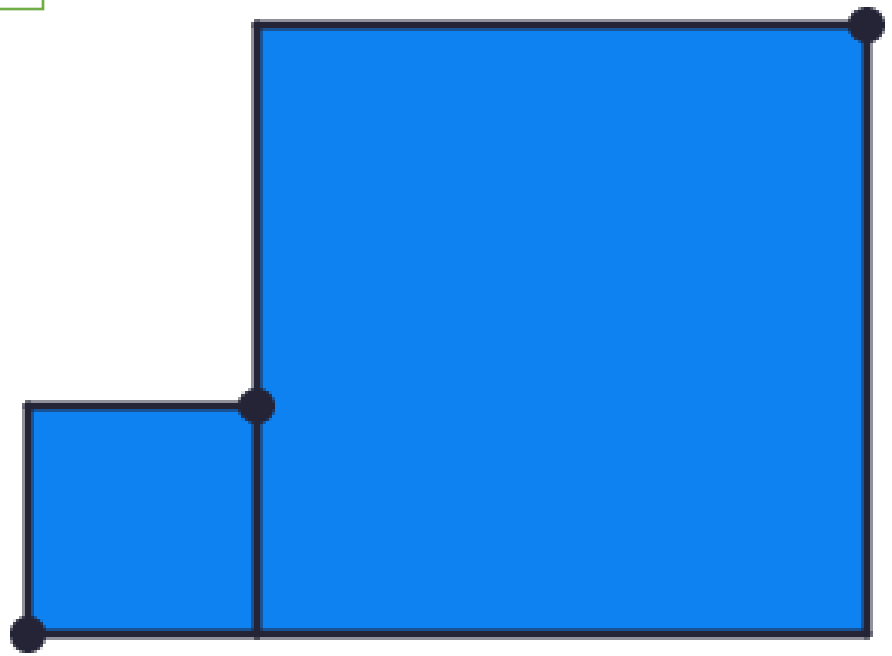
- engage with a dynamic geometry software package
- encounter geometrical results through **investigation** and **discovery**
- come to appreciate that certain features of shapes or diagrams appear to be independent of particular examples



# Student Activity 1A

What do you notice?



# Student Activity 1B

Investigate:

Create two squares on your geoboard

How many ways can you arrange two touching squares so that the dots are collinear?

**10 mins**

**Success Criteria:**

I can

- use my Geoboard to construct a square
- move the small square to find one set of collinear points
- identify an alternative location with collinear points
- find multiple solutions

**Extensions**

- find a new solution(s) if the squares do not need to be touching
- write a rule to describe how the location of my squares results in collinear points

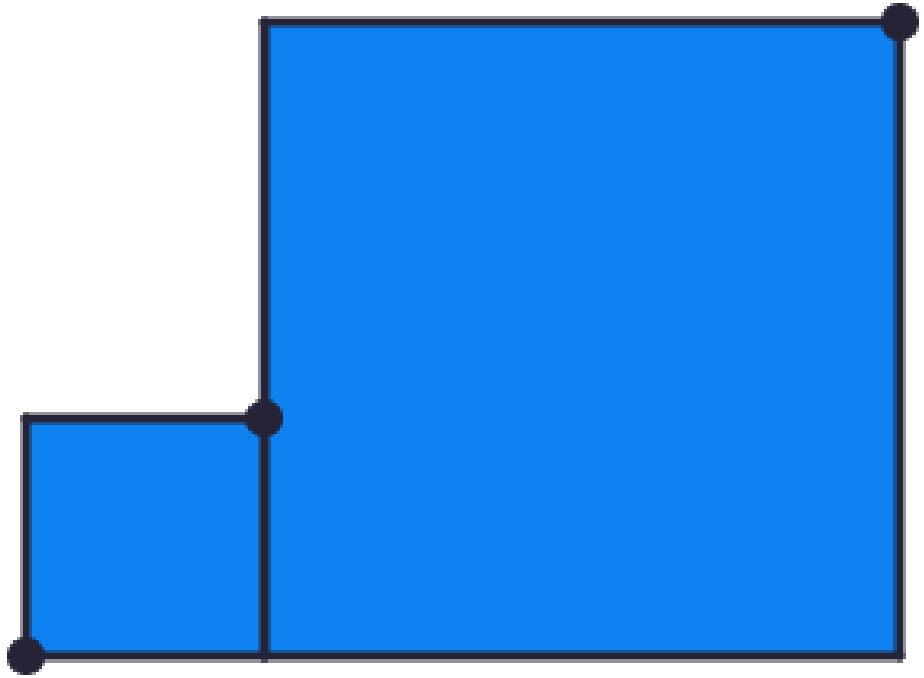
# Student Activity 1C

Investigate:

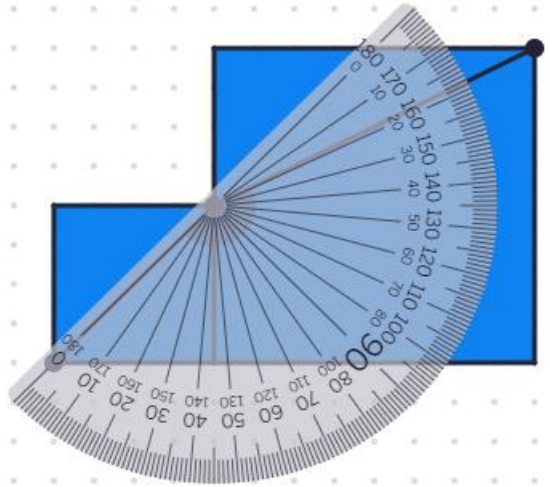
When the squares are side by side as shown in the diagram, is it possible for the dots shown on the vertices to be collinear?

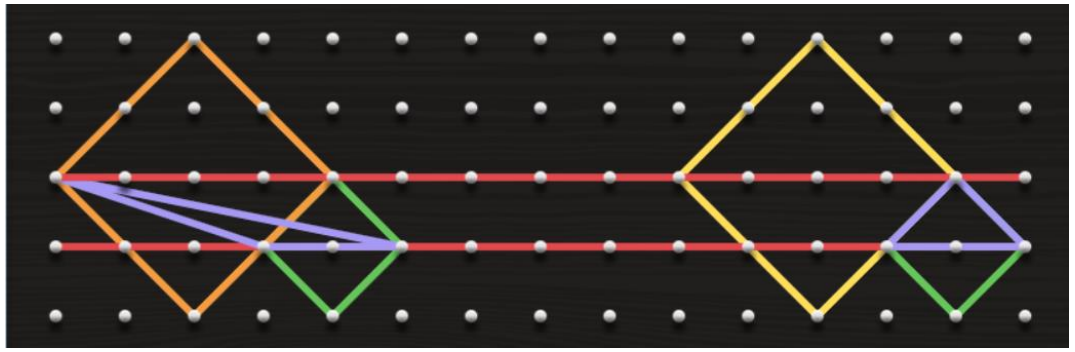
Justify your solution.

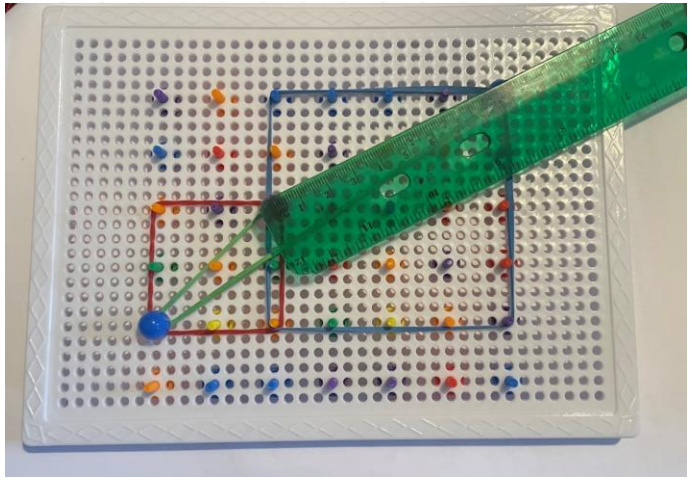
**10 mins**

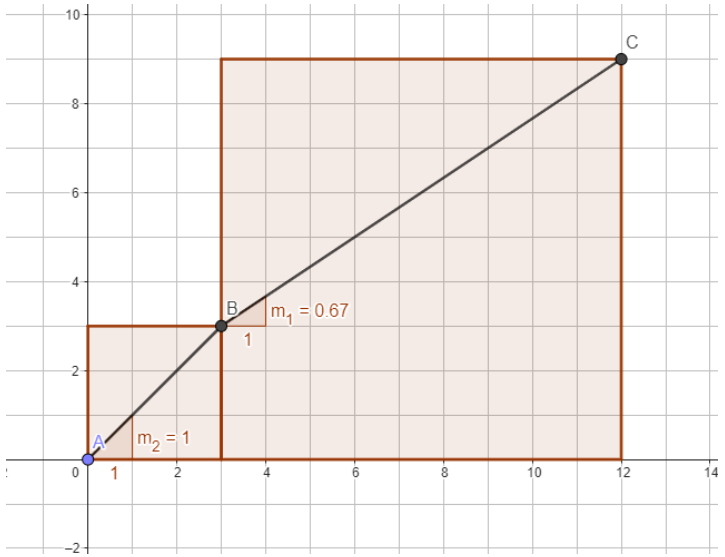


# Investigating Collinearity





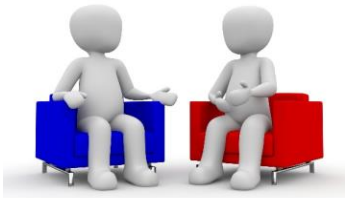




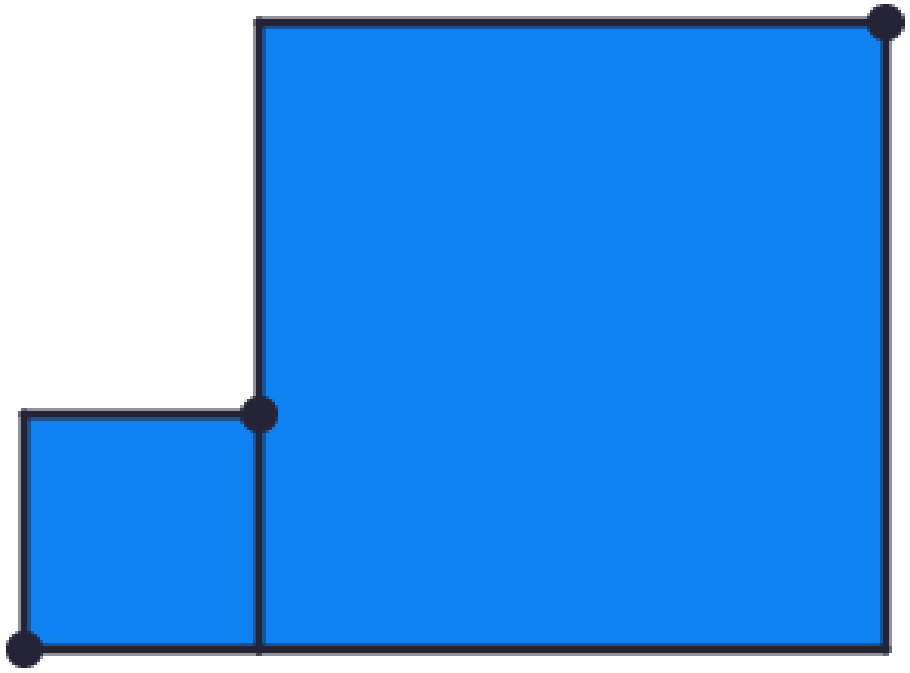
# Activity 1D

What happens if you vary the size of the squares?

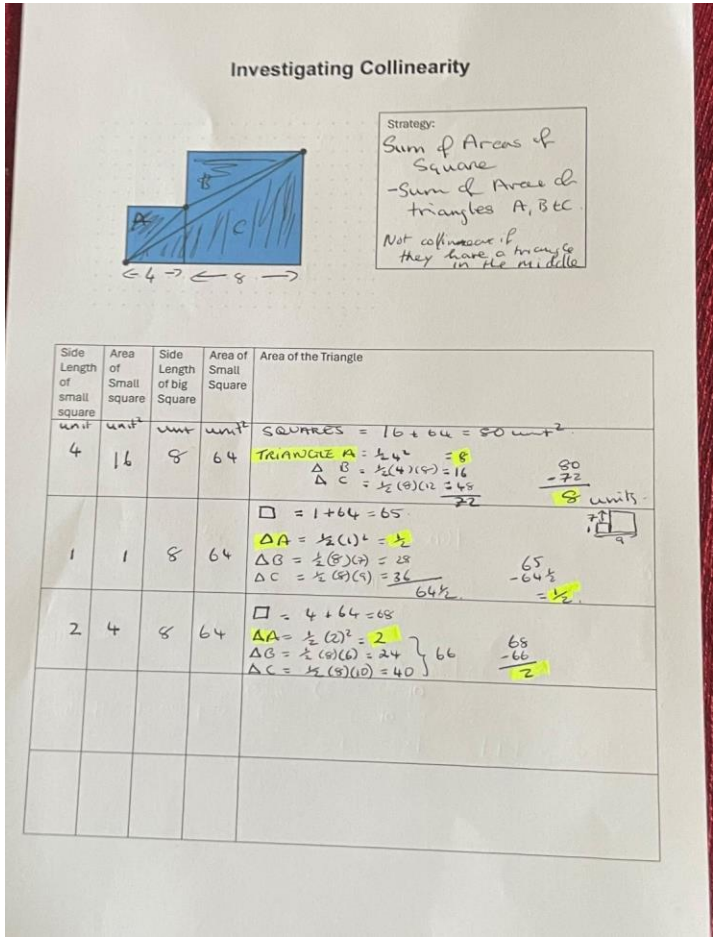
What do you notice?

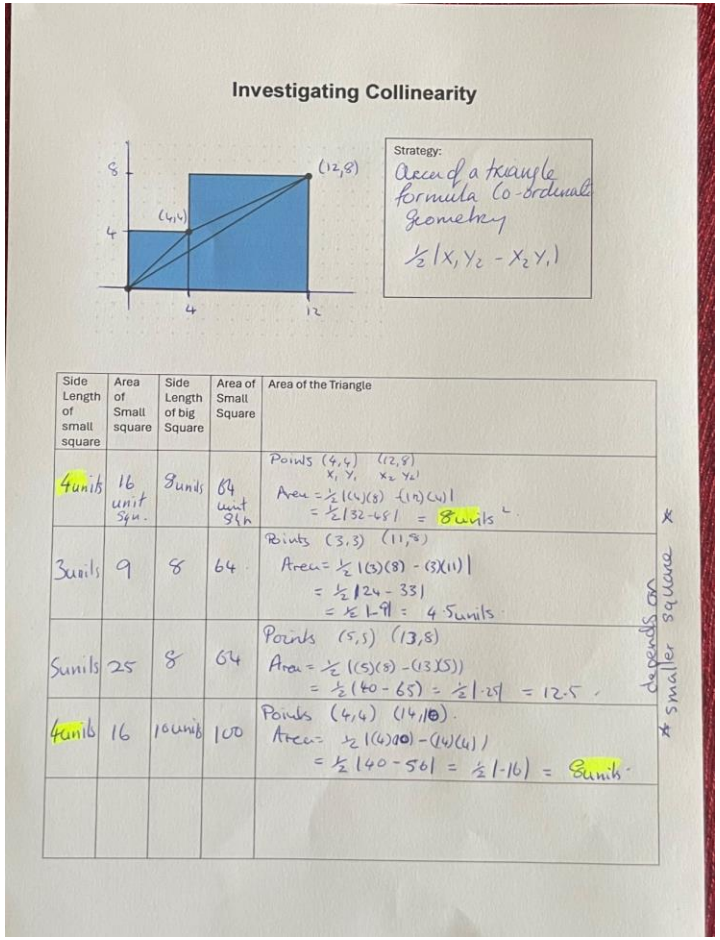


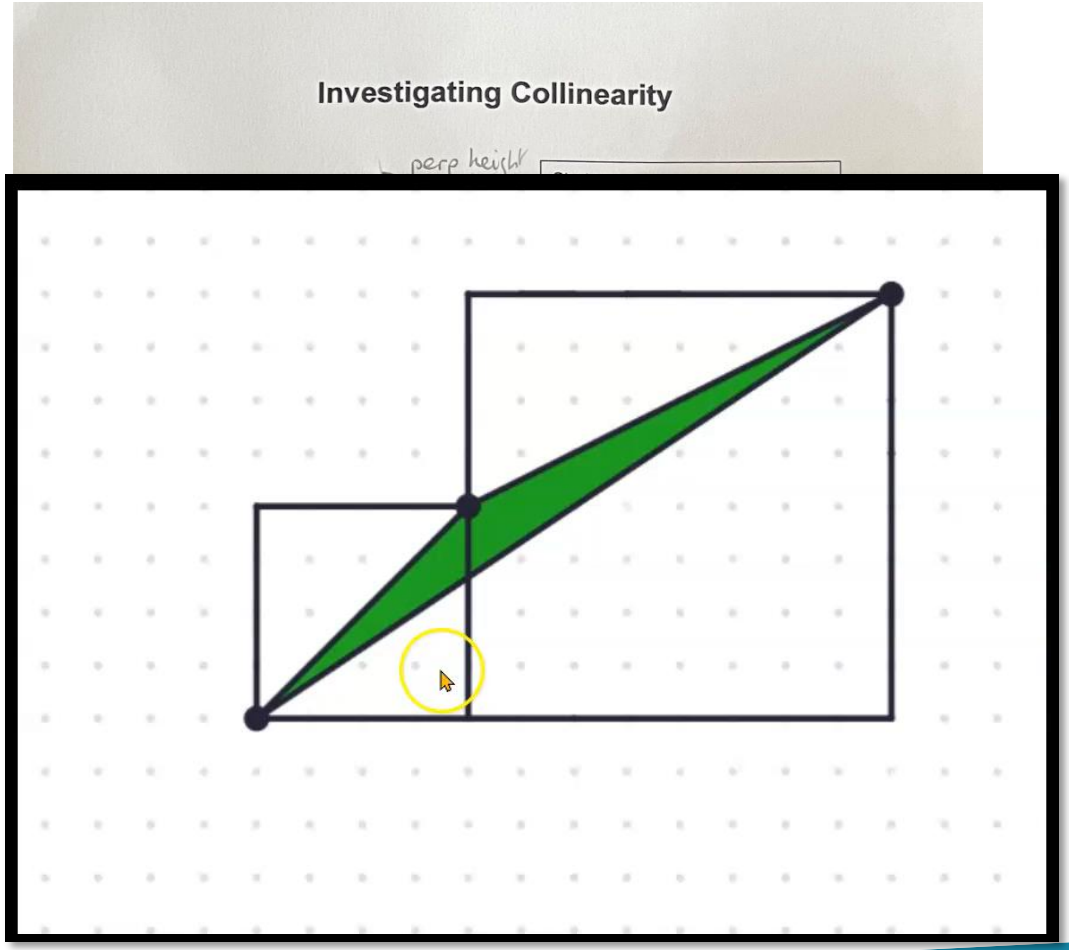
**15 mins**



# Possible Solutions



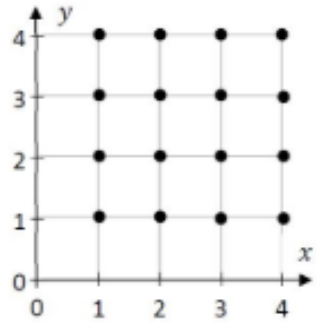



# Unfamiliar Context

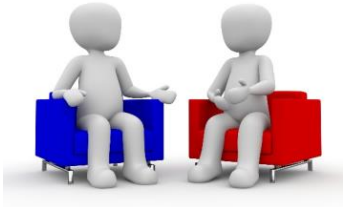
**Junior Cycle Sample Paper 2021**

In the co-ordinate diagram below, 16 points are marked with a dot (•).




**(a)** Louise picks 1 point at random from the 16 points marked with a dot in the diagram. She then finds the equation of the line that goes through this point and through (0, 0).
Find the **probability** that Louise's line has a slope that is **greater than** 1.

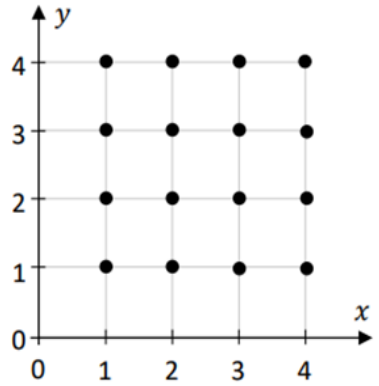
**10 mins**



**Leaving Cert Ordinary Level 2014 Paper 2**

In the co-ordinate diagram below, 16 points are marked with a dot (•).
These are all of the points of the form $(m, n)$, where $m, n \in \mathbb{N}$ and $m, n \leq 4$.

**Leaving Cert Higher Level 2024 Paper 2**




A pair of these points is picked at random.

**(i)** How many different pairs of points can be picked from these 16 points?

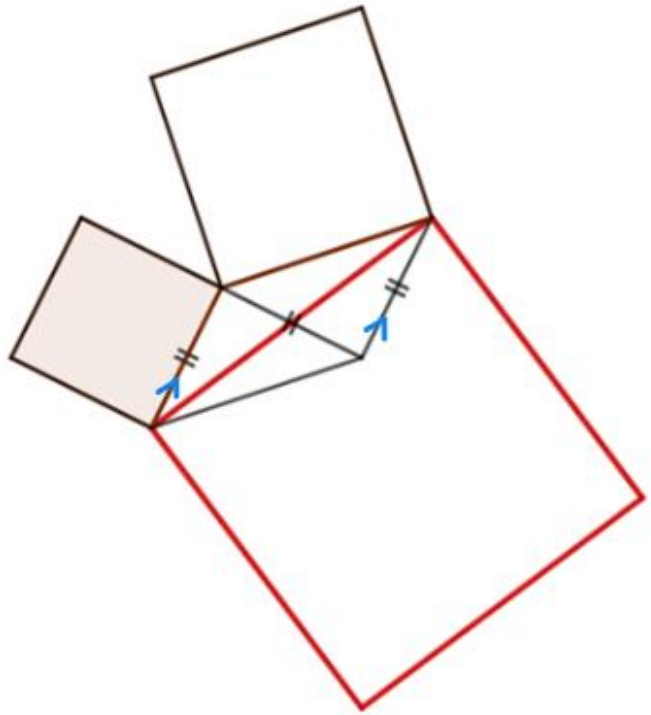
**(ii)** The two points that are picked are joined with a straight line.
Find the **probability** that this line is horizontal.

# Unfamiliar Context

**Task:**

Three squares are constructed as shown.

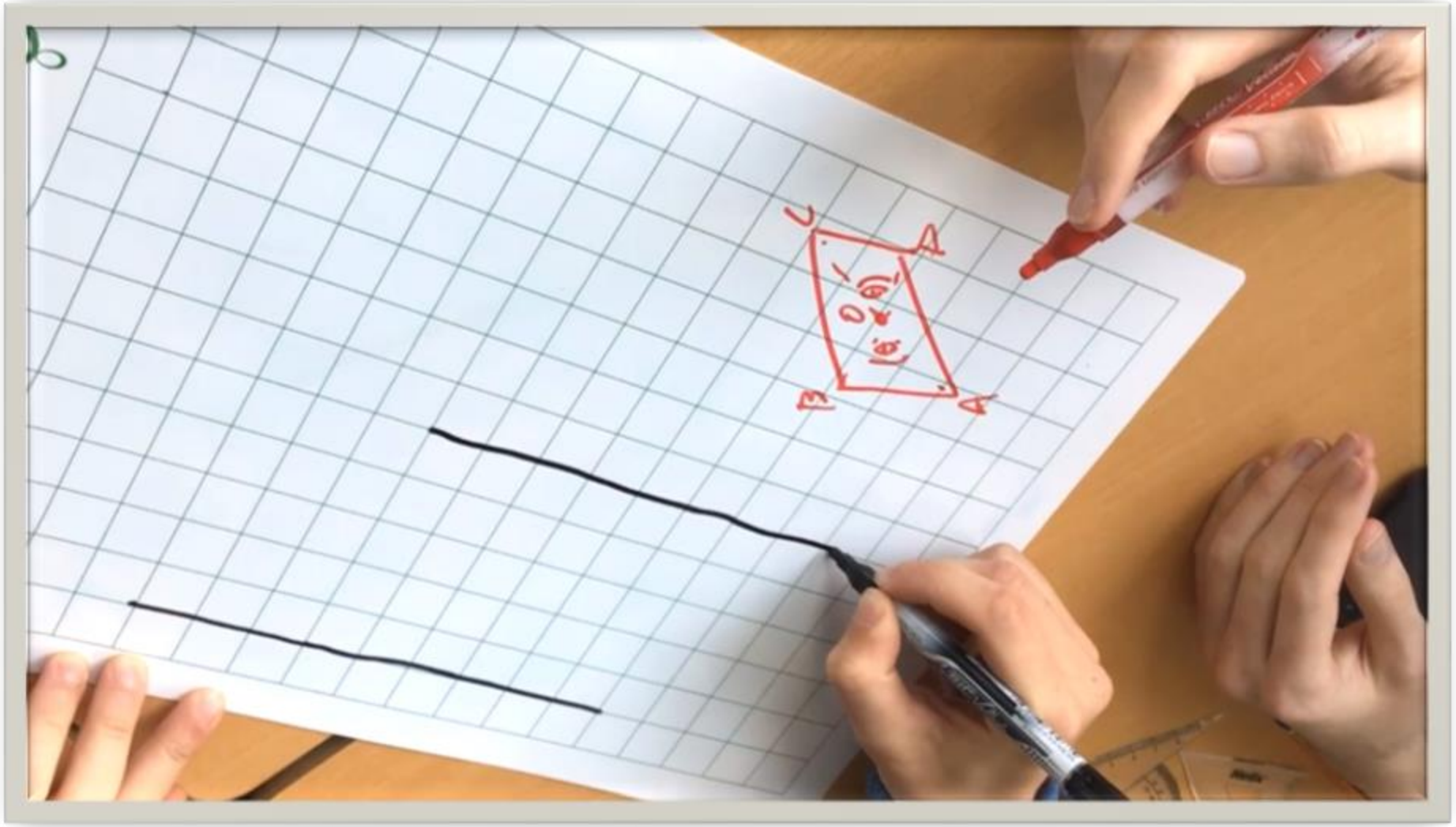
Prove that the area of the red square is five times the area of the shaded square.



**Success Criteria**

- I can sketch the diagram
- I can identify all relationships that I see on the diagram
- I can classify each of these relationships
- I can use the relationships to deduce that the area of the red square is five times the area of the shaded square
- I can formalise my proof
- I can generate more than one approach

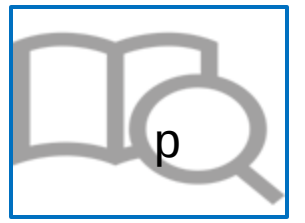
# Video - Unfamiliar Context



# Reflection

What was the key learning for you from this session?

How can you bring the learning to your classroom?



Oide

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Supporting the Professional Learning of School Leaders and Teachers

# Session 2

## Student exploration of geometric concepts

**Learning Intention:**

To consider how to develop students' geometric thinking.

**Success Criteria:**

I have

- collaborated to identify important concepts and activities students need to experience before engaging with formal proof
- explored how to support the development of my students' geometric thinking through choice of activity
- considered how the use of manipulatives supports students' engagement with geometry

# Discussion 2a

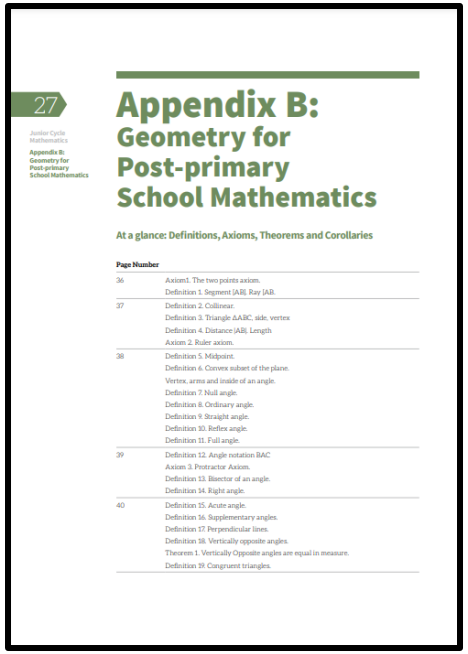
*'If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal'*.

Prior to engaging with this proof:

What concepts do students need to explore?

What activities should students have experienced?

**10 mins**



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Junior Cycle Mathematics

Appendix B: Geometry for Post-primary School Mathematics

## Appendix B: Geometry for Post-primary School Mathematics

At a glance: Definitions, Axioms, Theorems and Corollaries

| Page Number | |
|---|---|
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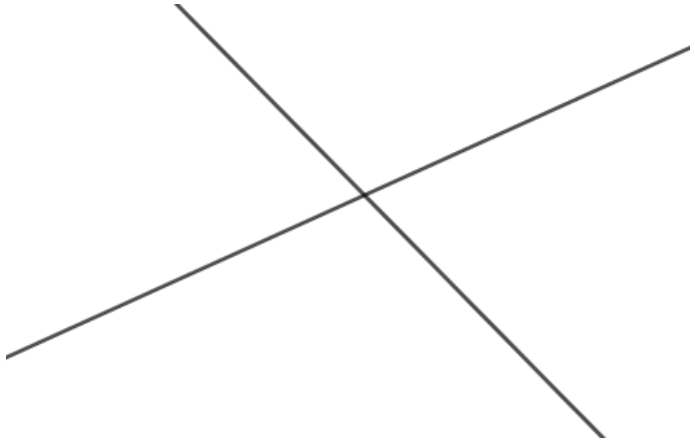
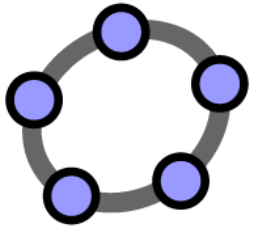
# Student Activity 2A

What do we notice?





**10 mins**

**Success Criteria**

I have

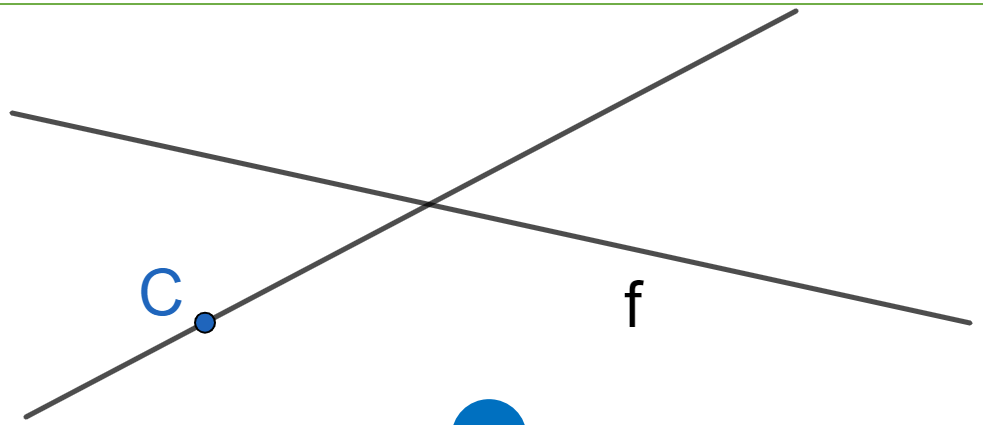
- measured one angle with my protractor.
- measured all four angles.
- identified a relationship between two angles
- identified another relationship between two angles
- used my geostrips to investigate a relationship

**Extension:** Write a rule to describe a relationship. Is this rule always/sometimes/never true?

# Student Activity 2B

Draw a line parallel to the line f through the point C.

What do you notice?




**10 mins**

**Success Criteria**

I can

- draw a line through the point C
- explain why my line is parallel to f
- identify angles on my diagram
- make a conjecture about two angles
- identify a new relationship to investigate

I have

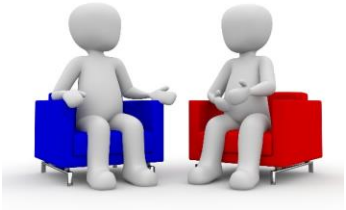
- used my protractor to investigate
- discussed my thinking with my group
- summarised my findings

**Extension:** Write a rule to describe a relationship. Is this rule always/sometimes/never true?

# Student Activity 3

Draw a triangle and **measure** all the dimensions.

Give your partner the minimum number of measurements needed to **construct** the same triangle?



**20 mins**

**Success Criteria**

I have

- used a straight edge to draw a triangle
- measured and labelled the dimensions
- generated a list of instructions
- constructed my partners triangle
- verified my partners triangle is identical
- discussed with my partner why my solution has the minimum number of instructions
- investigated with my partner if there is an alternative list of instructions for my triangle

**Extension:** Write a rule to describe the minimum numbers of instructions needed to create any triangle.

# Video - Activity 3

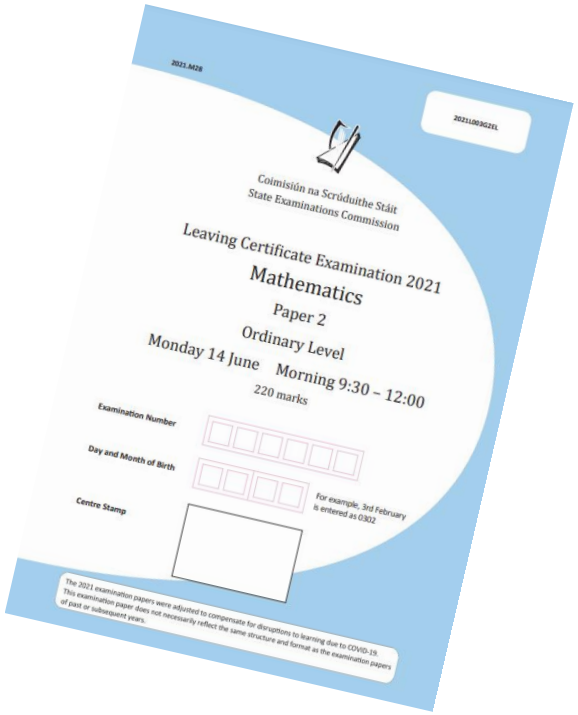
# Unfamiliar Contexts

**(a)** Construct the triangle $ABC$ such that $|AB| = 8$ cm, $|BC| = |AC| = 5$ cm. The point $A$ is given to you.

$A$ •——————————

**(b)** On the same diagram, construct the image of the triangle $ABC$ under the axial symmetry in $AB$.

**(c)** Justify the statement "$AC'BC$ is a parallelogram" where $C'$ is the image of $C$ under the axial symmetry in $AB$.



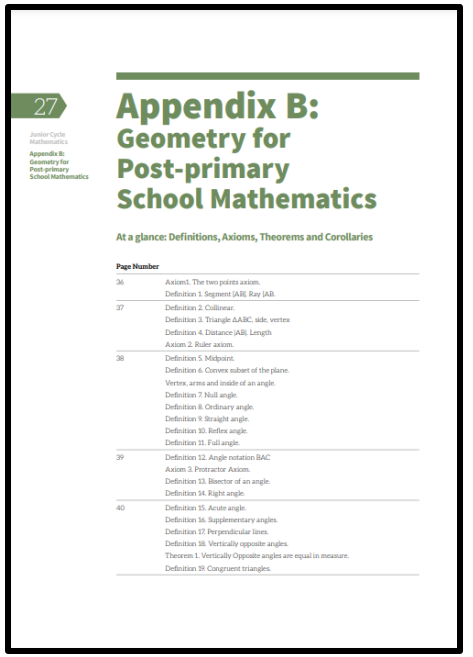

**Leaving Cert Ordinary Level 2013 Paper 2**

# Discussion 2b

*'If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal'.*

What additional activities would help support students before engaging with proving this.

**10 mins**



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Junior Cycle Mathematics

Appendix B: Geometry for Post-primary School Mathematics

## Appendix B: Geometry for Post-primary School Mathematics

**At a glance: Definitions, Axioms, Theorems and Corollaries**

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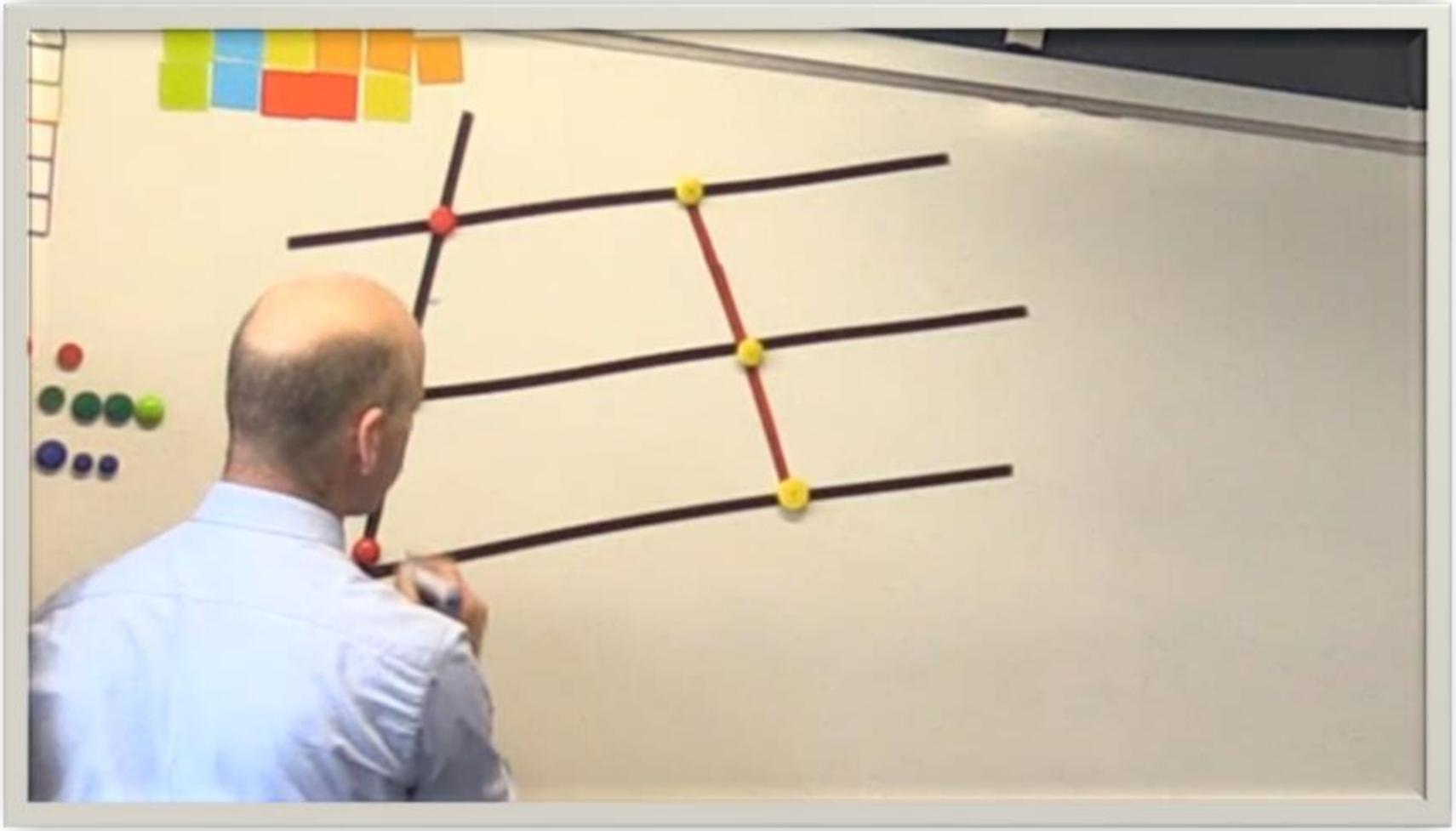
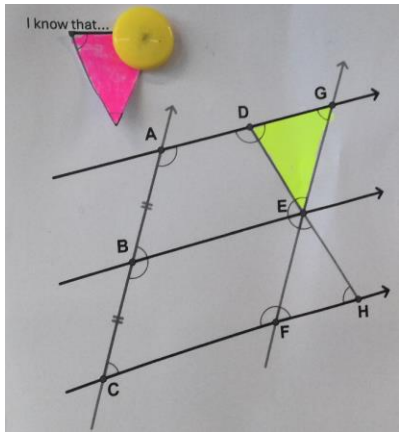
# Video – Activity 4

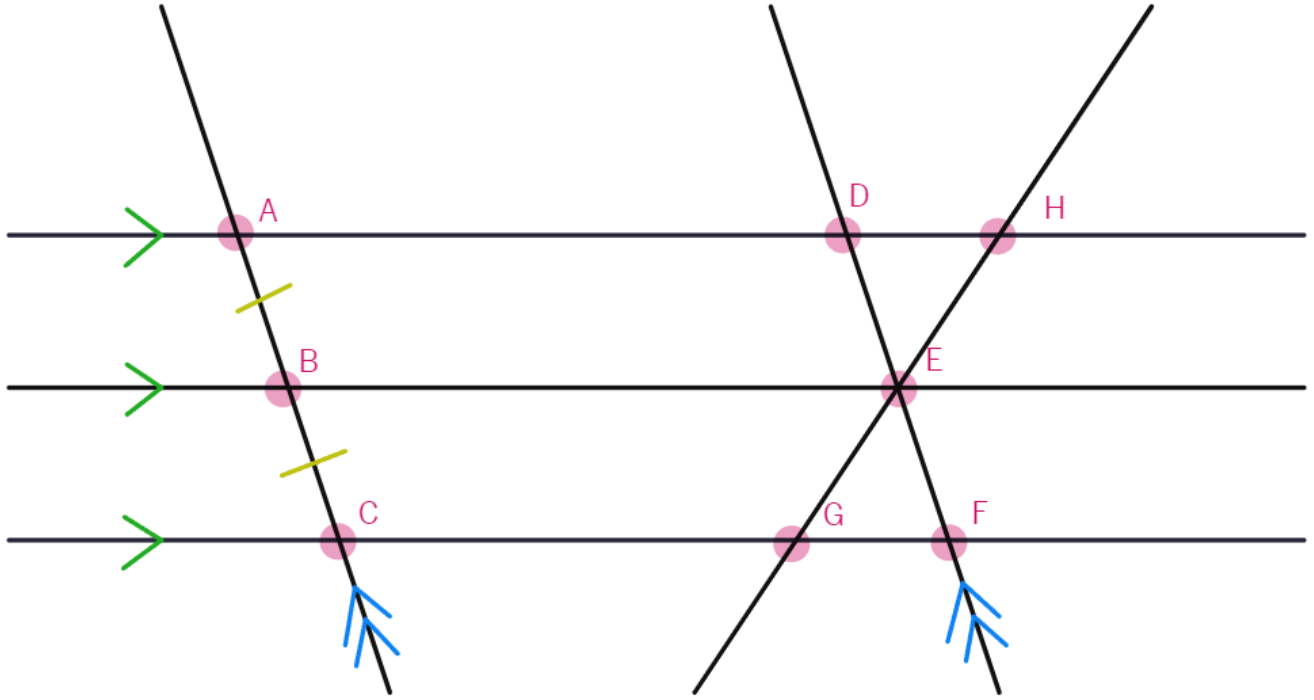




# Student Activity 4

Investigate:

Are the other two sides equal?




**Success Criteria**

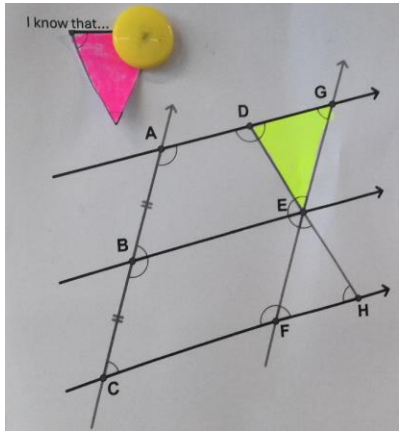
I have

- identified relationships on the diagram
- justified each of these relationships
- discussed my thinking with my partner
- provided a convincing argument to answer the question posed

**Extension:** Write a proof to support your investigation

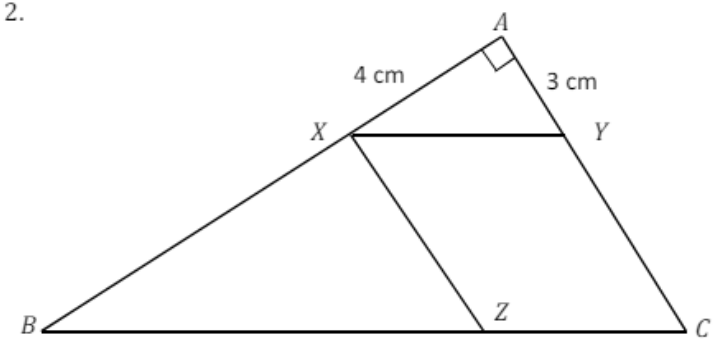
**10 mins**

# Video – Activity 4



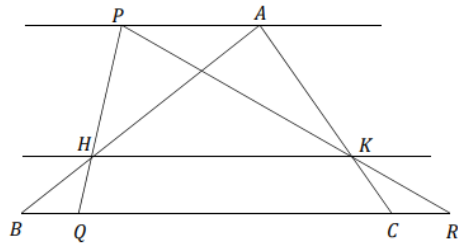
# Unfamiliar Context

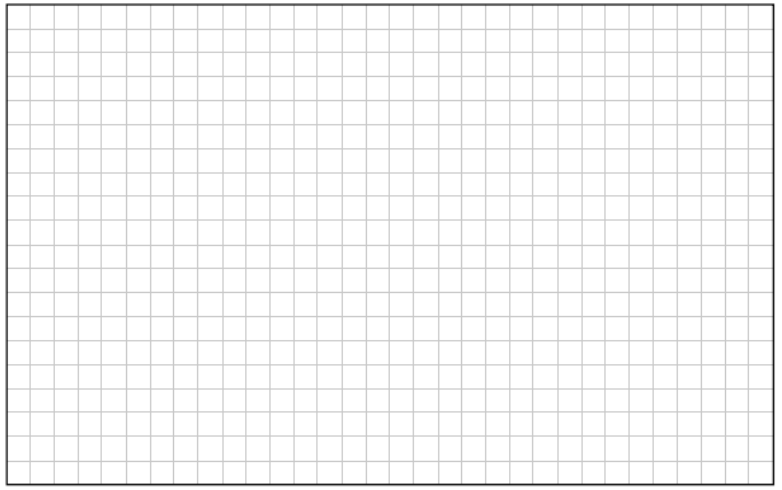
**(b)** In the triangle $ABC$ shown below:
$|\angle CAB| = 90°$, $|AX| = 4$ cm, $|AY| = 3$ cm, $XY \parallel BC$, $XZ \parallel AC$,
and $|AX| : |XB| = 1 : 2$.
Find $|BZ|$.




**Leaving Cert Higher Level 2018 Paper 2**

**(b)** In the diagram below, the lines $PA$, $HK$, and $BR$ are parallel.
Prove that $|AH| \times |QB| = |AP| \times |HB|$.
Give a reason for each geometrical statement you use.

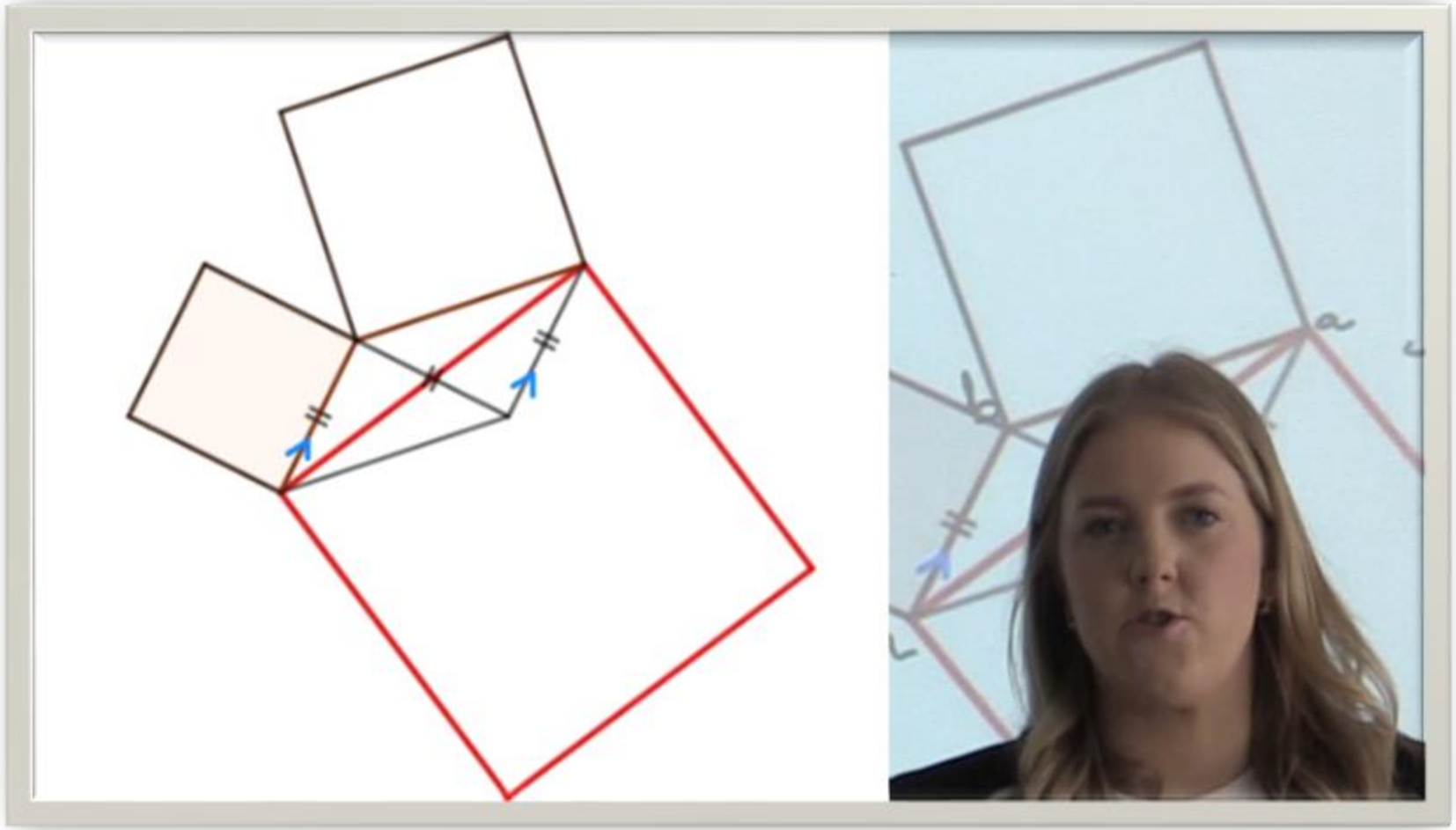


**Leaving Cert Higher Level 2021 Paper 2**

# Video – Student Voice

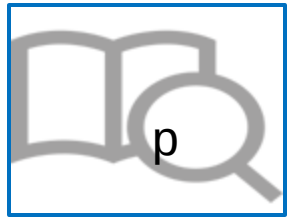
# Video – Teacher Voice



# Reflection

What was the key learning for you from this session?

How can you bring the learning to your classroom?



Oide

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# Session 3

## Designing learner experiences

**Learning Intention:**

To increase opportunities for meaningful student engagement with geometrical concepts

**Success Criteria:**

I have

- worked with a group to choose/create an activity
- created supporting activities for diverse learners
- reflected on the learning from this event

I can

- create learning experiences that will support the development of my students' geometric thinking

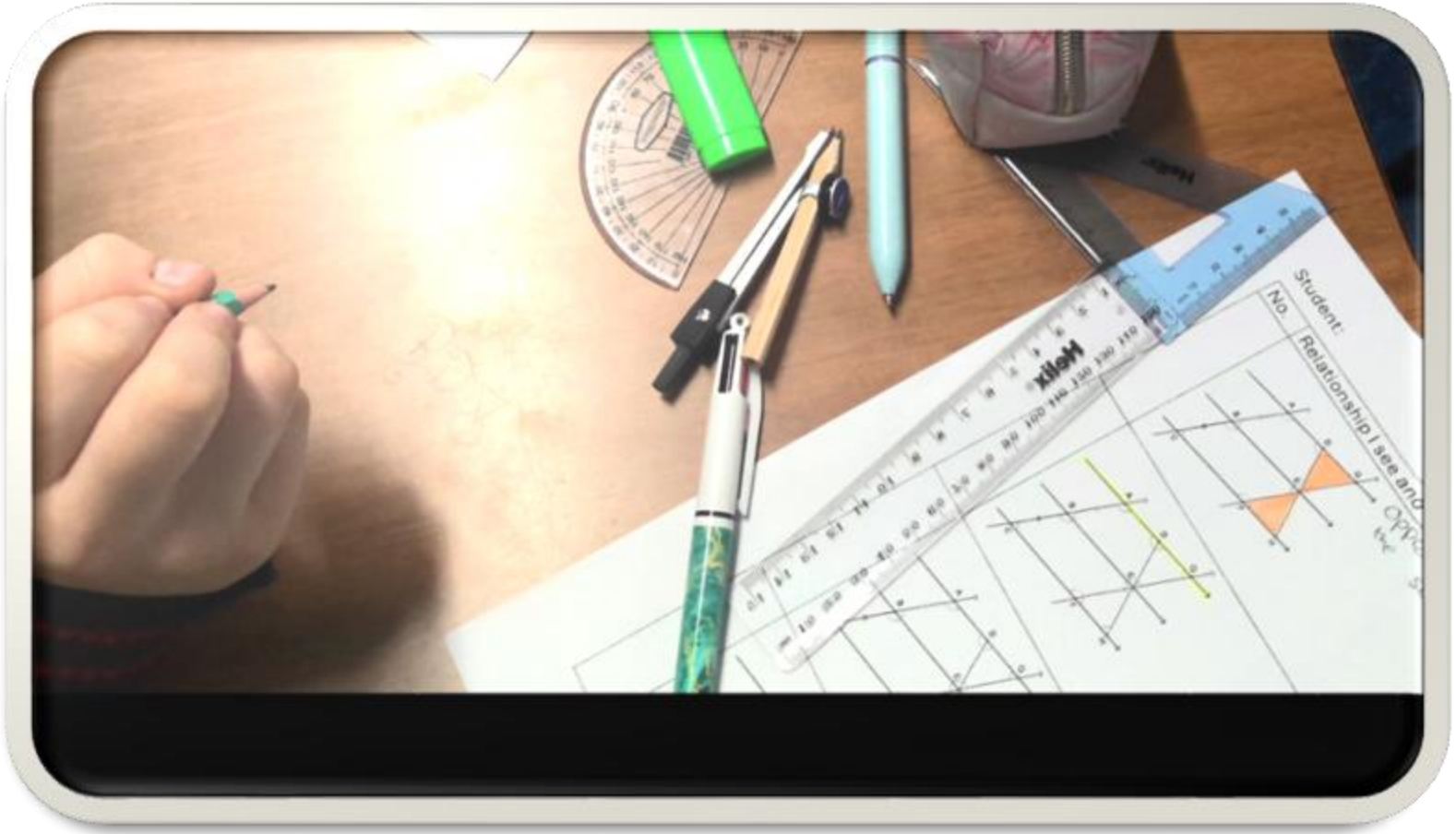
# Discussion 3

How are our students currently engaging with Geometry?

When should students encounter Geometry to ensure they are best supported to develop their geometric thinking?

**10 mins**

# Teacher Voice



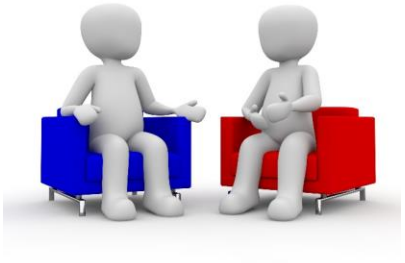
# Design a Learner Experience

Identify a concept to explore involving quadrilaterals.

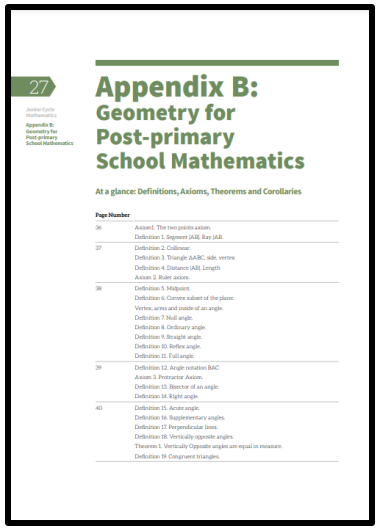
**Success Criteria**

I have

- discussed the concepts within the quadrilaterals section located within the appendix
- identified one concept to explore within this topic
- chosen a student group and identified the context



**10 mins**



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Appendix B: Geometry for Post-primary School Mathematics

At a glance: Definitions, Axioms, Theorems and Corollaries

# Van Hiele Levels

| | **Supporting Activities** |
|---|---|
| **Level 0 (Visualisation)** | Sorting shapes<br>Find a rule<br>Physical manipulatives |
| **Level 1 (Analysis)** | Property lists<br>Class lists |
| **Level 2 (Informal Deduction)** | Informal logical reasoning tasks<br>Digital manipulatives |
| **Level 3 (Deduction)** | Explore abstract statements |

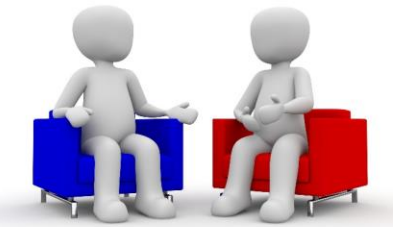
# Design a Learner Experience

Create a learner experience to explore quadrilaterals.

**Success Criteria**

I have

- designed an activity to support engaging with our chosen concept
- produced supporting activities to support our diverse learners
- included the use of digital and physical manipulatives
- created a presentation describing my learner experience



**Extension:**

I have

- made links to a construction(s)
- made connections with other concepts within the syllabus/specification
- included a real-world situation to explore

**30 mins**

# Sharing our thinking

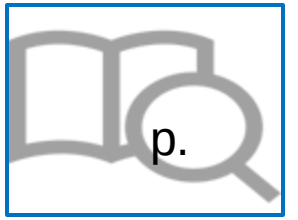
How have you engaged students with Geometry to develop conceptual understanding?

How could you adapt this task to meet the needs of your own students?

# Reflection

What was the key learning for you from this session?

How can you bring the learning to your classroom?



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# Conclusion

# Learning Intentions

- To consider how to develop students' geometric thinking

- To increase opportunities for meaningful student engagement with geometrical concepts

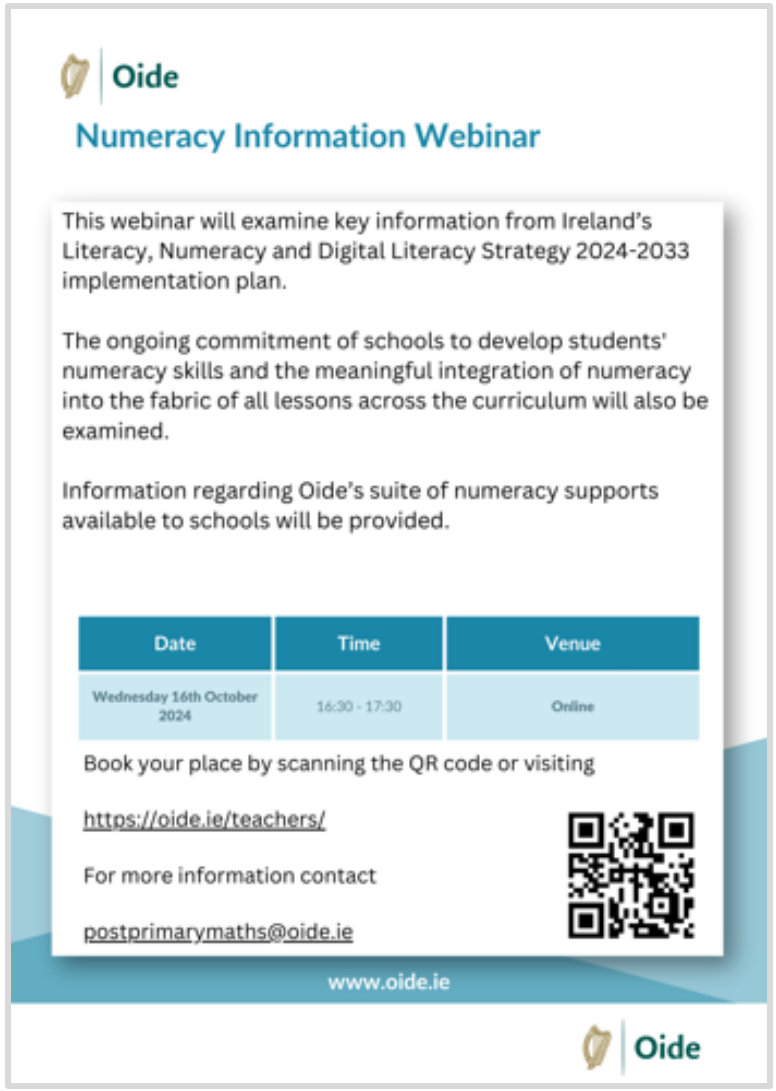
# Key Message

Students need exposure to active learning to develop geometric thinking.

# Feedback

https://registration.oide.ie/feedback

# Upcoming Events - Numeracy webinar



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