An Roinn Oideachais
agus Scileanna

Applied Mathematics

Guidance to support the completion of the Modelling Project

LEAVING CERTIFICATE
Ordinary and Higher Level

Contents

1 Introduction

This document, *Leaving Certificate Applied Mathematics: Guidance to support the completion of the Modelling Project for Leaving Certificate Applied Mathematics* provides

- detail of the nature and scope of the Modelling Project, described in the curriculum specification for Leaving Certificate Applied Mathematics
- guidance and support for schools, teachers and students on completing the Modelling Project.

These guidelines should be used in conjunction with the curriculum specification for Leaving Certificate Applied Mathematics[1].

The State Examinations Commission is responsible for the development, assessment, accreditation and certification of the second-level examinations of the Irish state. A common brief which contains instructions and clarification to all examination candidates of the procedures for completion and submission of the Modelling Project Report, can be found on the SEC website at www.examinations.ie.

1 www.curriculumonline.ie/getmedia/1d61d7b6-573d-4e2a-83ea-037ef17b083b/Leaving-Certificate-Specification-Applied-Mathematics_EN.pdf

2 Assessment for Certification in Applied Mathematics

Assessment for certification is based on the aim, objectives and learning outcomes of the Leaving Certificate Applied Mathematics specification.[2]

Assessment components

There are two assessment components in Leaving Certificate Applied Mathematics:

- written examination (80%)
- modelling project (20%)

Both components of assessment reflect the relationship between the application of skills and the theoretical content of the specification. Differentiation at the point of assessment is achieved through written examinations at two levels – Ordinary level and Higher level. The modelling project will be based on a brief issued annually by the State Examinations Commission (SEC). A common brief will be issued for Ordinary level and Higher level. A differentiated marking scheme will apply.

Table 1: Overview of assessment

MODE	TIMING	FORMAT	WEIGHTING AT ORDINARY LEVEL	WEIGHTING AT HIGHER LEVEL
Written Examination	End of year 2	Answer booklet	80%	80%
Modelling Project	Year 2	Report	20%	20%

2 https://www.curriculumonline.ie/Senior-cycle/Senior-Cycle-Subjects/Applied-Mathematics/

3 The Modelling Project

The modelling project assessment will require students to demonstrate proficiency in course content and skills that cannot be easily assessed by the written examination. The assessment will require students to present a solution to an authentic modelling problem, and to report on the work and process involved. Students must acknowledge (i.e. through citation, through attribution, by reference, and/or through acknowledgement in bibliographic entry) the source or author of all information or evidence taken from someone else's work. Student work will be submitted to and marked by the State Examinations Commission (SEC).

Through the modelling project, students are afforded an opportunity to engage in the full modelling cycle to propose a solution to an authentic problem in a real context. The modelling project will assess the student's ability to use mathematics to represent, analyse, make predictions or otherwise provide insight into a real-world phenomenon. The key skills of processing data and information, communicating, critical and creative thinking, being personally effective and working with others can be developed through all the learning in this course, and these skills will be applied through the student's engagement in the modelling project.

The modelling project will be based on a brief issued annually by the State Examinations Commission (SEC). A common brief will be issued for Ordinary level and Higher level. The brief will outline a modelling problem in a real-world scenario. There is no pre-determined solution strategy and the students have ownership of all decisions they make as they progress through the modelling cycle to arrive at their solution. The brief will also outline the parameters for the problem and for the format of the report, which will be submitted to the SEC for assessment. The modelling project will be completed in sixth year.

The modelling project requires students to demonstrate that they can:

- define a problem
- translate the problem to mathematics
- compute a solution
- analyse the solution and iterate the process.

The report must be the student's own work. Authentication procedures will be in place to ensure compliance with this requirement. These will include a protocol in relation to the use of internet-sourced material.

4 Mathematical Modelling

In the unifying strand students learn about mathematical modelling as a process that uses mathematics to represent, analyse, make predictions or otherwise provide insight into real-world phenomena. The process is iterative and translates between the real world and mathematics in both directions and involves a number of stages.

The specification promotes a modelling first approach to solving authentic practical problems and students should have developed sufficient knowledge, skills and understanding over the duration of the course to undertake a Modelling Project in 6th year. As part of ongoing teaching, learning and assessment of the learning outcomes for Leaving Certificate Applied Mathematics, students should have opportunities to develop the mathematical modelling competency as they realise various learning outcomes across strands, as appropriate, including asking questions about the world around them, simplifying problems; decomposing them into manageable parts, stripping away any superfluous information and analysing the situation for structure and similarities to other problems, using appropriate assumptions, mathematising situations and interpreting the solution to problems in context.

As students' progress through senior cycle, they should be encouraged to identify questions within mathematics and from other subjects or the world around them that they want to know more about. While the Modelling Project assessment is summative, it is envisaged that throughout the two years of senior cycle, formative assessment by teachers, the students themselves and their peers is used to allow students, and teachers to aid their development and track their progress. From an early stage, through their engagement with mathematical modelling problems[3], students should be familiar with the modelling cycle set out as learning outcomes in the unifying strand.

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3 *Modelling problems require the solver to research the situation themselves, make reasonable assumptions, decide which variables will affect the solution, and develop a model that provides a solution that best describes the situation*
LC Applied Mathematics specification page 10

Table 2: Mathematical modelling

STUDENTS LEARN ABOUT	STUDENTS SHOULD BE ABLE TO
The problem-solving cycle	‣ describe a systematic process for solving problems and making decisions
Formulating problems	‣ research the background to a problem to analyse factors or variables that affect the situation ‣ determine information relevant to the problem ‣ decompose problems into manageable parts ‣ determine what assumptions are necessary to simplify the problem situation
Translating problems into mathematics	‣ use abstraction to describe systems and to explain the relationship between wholes and parts ‣ abstract the knowledge needed to build a mathematical model ‣ translate the information given in the problem together with the assumptions into a mathematical model that can be solved
Computing solutions	‣ compute a solution using appropriate mathematics ‣ create a mathematical model that can be interpreted by a computer ‣ use computational technology to solve problems ‣ solve the mathematical problem stated in the model ‣ analyse and perform operations in the model ‣ interpret the mathematical solution in terms of the original situation
Evaluating solutions	‣ refine a model and use it to predict a better solution to the problem; iterate the process ‣ communicate solution processes in a written report

5 Process for Completing a Modelling Project

Over the course of no more than 20 hours of in school time, students will engage in activities that belong to 4 distinct stages of the mathematical modelling cycle. These activities contribute to the generation of their evidence of learning and achievement in the Modelling Project.

1. Formulating problems
2. Translating problems into mathematics
3. Computing and interpreting solutions
4. Evaluating and reporting solutions.

It is not intended to present the stages as a rigid and linear process. Modelling problems by their nature are messy and complex and it is not possible to simply move through the stages consecutively to produce an answer. Students should be prepared to iterate the process; that is to move backwards and forwards between the stages and revisit activities at different times as they complete the investigation. An online resource suitable for all abilities exemplifies the guidance provided in this section and is outlined in Appendix 1.[4]

1. Formulating Problems

This part of the investigation involves students engaging with a brief which provides information about a phenomenon from the world around them. The brief is open-ended, it does not include a straight forward question presenting an opportunity for creative problem-solving and interpretation. On reading the brief students have to be able to decide what exactly is the problem they would like to investigate. Is it a question they would like to seek to answer or is there a particular aspect of the phenomenon they would like to shed some light on?

Once a student has decided the problem they would like to investigate, they will need to define the problem. This process of understanding and exploring the problem is important planning and requires not only knowing what they want to find but also the key pieces of information that need to be put together to obtain a solution to the problem.

Real-world problems are broad and complex and during this stage of the modelling process students refine their idea; the goal being a concise problem statement that indicates exactly what the output of their mathematical model will be. To do this they will need to do some research and brainstorming to decide what are the main factors influencing the phenomenon and what factors can be quantified. They may also find it necessary to make certain assumptions that help to simplify the problem and sharpen the focus.

4 https://www.wolframcloud.com/env/CBM/TeachingPlatform/WelcomePage.nb

2. Translating the problem to mathematics

Once students have a problem statement clearly defined they are ready to embark on the next stage of the process converting the precise question to a mathematical form. Now they draw on their research and prior learning, making connections within mathematics, with mathematics and other subjects or with mathematics and the world around them. During this phase of the process they interpret what is needed to solve the problem so that they can determine how to find the answer.

It is impossible to account for all the important factors in a given situation, there may be variables or relationships for which data is unavailable and in these cases students must make choices about what to include in their representation of the real-world. Making assumptions helps reveal the variables that will be considered and also reduce the number of them by deciding not to include everything. Within this process, relationships between variables will emerge based on observations, physical laws, or simplifications leaving students ready to translate the problem to mathematics by defining the details of their mathematical model. They then use the representation they devise to analyse the problem situation mathematically, draw conclusions, and assess them for reasonableness of the solution.

3. Computing and interpreting solutions

Once students have decided on a strategy or an initial mathematical representation of the phenomenon they can use a mathematical technique to get a solution to the problem. This part of the process can be exciting for students as it is at this stage that they get the first glimpse of their results. Getting the solution involves students looking into their personal toolkit for a mathematical technique to use. This stage of the process involves creativity and an understanding that different solution strategies can lead to solutions of a different nature.

Having found a solution students must decide whether their answer makes sense in relation to the real-world problem. It is during this sense making check that students see the need to iterate; revisit their assumptions, or look again into their mathematical toolkit to see if a different solution better addresses their problem.

The following questions can be used by teachers to support students through this stage of the process:

- Have you seen this type of problem before? If so, how did you solve it? If not, how is this problem different?
- Do you have a single unknown, or is this a problem with lots of variables that may depend on each other?
- Is the representation linear or non-linear?
- Are there any digital tools you could use to help?
- Would a graph or other visual representation help provide insight?
- Is your mathematical representation too complicated? How about you look again at your assumptions and try to simplify it?
- Can you hold some values constant and allow others to vary to see what is going on?

4. Evaluating and reporting

This stage of the process involves students assessing the quality of their model, they will only know if their model works when they start plugging numbers in and considering whether the theoretical value provided by the model is actually practically viable. The following questions can be used by students to evaluate their model

- Does my answer make sense?
- Is the sign of the answer correct?
- Is the magnitude of the answer reasonable?

If the answer doesn't make sense students should consider whether a mistake has been made in implementing their model

Another important consideration is whether or not the model behaves as expected? If the output of a model is visualised with a graph or plot of any kind, then carefully looking at features such as intercepts, the maximum or minimum values, or the long-term behaviour can help answer that question. If students have a data set and believe there is a relationship between two variables, then plotting the data can be insightful in assessing the behaviour of the model and considering the need to iterate.

Students should attempt to validate their model, using historical data with their model can give students insights into whether or not their model is doing what they want it to do. For example if a student's model of the An Post cycle race suggests they would complete the stage faster than a professional cyclist it may be necessary to iterate the process to revisit assumptions or to include other factors in the model.

Students should consider the model's sensitivity to changes in the assumptions and parameters used to build it.

6 The Modelling Project Report

Students are required to present and submit a Modelling Project Report digitally using the template, file format type and instructions specified by the State Examinations Commission (SEC). The completed report will comprise of some or all of the following elements: written text, data tables, diagrams, digital images and photographs. All images must be captured, edited and published in accordance with the requirements of the school's Acceptable Use Policy (AUP), Data Protection Policy, and General Data Protection Regulations (GDPR) in the booklet provided by the State Examinations Commission.

A common brief for all students studying Leaving Certificate Applied Mathematics will be issued by the State Examinations Commission after mid-term in year 2. A common brief is issued to account for the fact that at the time of undertaking the project and writing up the Modelling Project Report students may not yet have decided what level of written paper they will take. Differentiation in marking will take place after students have indicated their level of examination entry. The Report must be presented in a digital completion booklet. Students will be expected to spend no more than 20 hours completing the modelling project, this must be done in school. The completion date for submission of the Modelling Project Report will be close to mid-term in February of year 2; the precise date will be set by the State Examinations Commission via a circular. The table below provides an overview of the main sections and indicative content that may be included in the report.

Table 3: Overview of the main sections and indicative content that may be included in the report

SECTION	INDICATIVE CONTENT
Introduction and Research	▸ Background research on brief including citations and references ▸ Defining the specific problem(s) to be modelled ▸ Research on the specific problem(s) including citations and references ▸ Identification of the relevant variables ▸ Presentation of relevant data
The Modelling Process	▸ Explanation and justification of the model and assumptions ▸ Computation of the solutions ▸ Presentation of solutions using appropriate mathematical and graphical representations ▸ Analysis of solution(s) – sensitivity to changes in assumptions; comparison with other solutions or real-world data ▸ Evidence that the process has been iterated
Interpretation of Results	▸ Interpretation of solution(s) in real-world context ▸ Conclusions and reflections
Communication and Innovation	This is not a distinct section of the report. ▸ Innovative and creative approaches ▸ Quality and clarity of presentation

Assessment Criteria for the Modelling Project

THE STUDENT DEMONSTRATING A HIGH LEVEL OF ACHIEVEMENT:	THE STUDENT DEMONSTRATING A MODERATE LEVEL OF ACHIEVEMENT:	THE STUDENT DEMONSTRATING A LOW LEVEL OF ACHIEVEMENT:
states the problem statement concisely, early in the written report. References sources from background research. identifies several variables affecting the model and notes and justifies the need for the main factor that influences the phenomena being modelled. clearly identifies and justifies the assumptions used to develop the model and, where appropriate, states the limitations of the simplification of the problem due to the assumptions made. indicates exactly what the output of the model will be and, if appropriate, identifies the audience and/or perspective of the modeller.	identifies a problem statement which is not precise or consistent with other statements in the report. lists important parameters and variables properly, but without sufficient explanation. notes primary assumptions, but without justification.	presents a problem statement that is difficult to understand or is buried in the text. identifies assumptions and justifies them, but they are difficult to identify in the text. barely mentions variables/ parameters or, if mentioned, they are difficult for the reader to identify in the text.
provides clear insight with logical mathematical reasoning into the mathematical method(s) used to describe the relationship between the variables, and to solve the problem. Presents a plausible approach and outcome.	states a mathematical approach, however with aspects of the method(s) which are inconsistent, difficult to understand or incomplete.	states a model which contains fixable mathematical errors.
clearly presents an accurately-computed solution and analysis of the relationship between variables, supported where appropriate with visual aids and graphic representation that is consistent with the original problem statement.	states an answer, however with aspects of the solution(s) which are inconsistent, difficult to understand or incomplete (e.g. fails to identify units of measure).	states an answer but without contextual background (i.e. appropriate graphics, appropriate units, etc.).
addresses the viability and reliability of the mathematical modelling solution. considers how sensitive the model is to changes in parameter values or altered assumptions; how it compares to other solutions or historical data. The model is refined and the process iterated.	addresses the viability and reliability of the mathematical modelling solution, however with analysis which lacks proper dimensionality, e.g. obvious consequences of the stated outcome are ignored or well-known comparisons are disregarded.	provides some analysis but without any sense of perspective. uses incorrect mathematics in the analysis.
presents a paper that is well-formatted and enjoyable to read, with easy to interpret visual aids (if appropriate).	presents a paper with multiple spelling, formatting or grammatical errors, visual aids which are missing key readability features or which do not clearly connect to the solution.	presents a paper with significant disregard for common spelling, grammatical and mathematical rules.

Differentiation in the Modelling Project

In the case of the report on the modelling project, differentiation will be effected at the point of assessment through the application of separate Higher and Ordinary level marking schemes. The scheme to be used will be determined by the level at which the candidate takes the written examination.

The Teacher's Role

The teacher has an important role to play in supporting and supervising the student. The most crucial role a teacher can play in preparation for the Modelling Project is to ensure students are facilitated in realising the learning outcomes of all four strands of the specification. This should be done in as many contexts as possible over the duration of the course and through consistent engagement with mathematical modelling problems. Engagement with the modelling cycle of Strand 1 is pivotal to students' readiness to carry out their project.

To facilitate the provision of feedback to students during their engagement with assessment, the process of completing the Modelling Project should be viewed as part of teaching and learning, and not solely for assessment purposes. It is envisaged that teachers will guide, support and supervise throughout the process. Support may include:

- Clarifying the requirements of the modelling project.
- Prompting the student's critical thinking in relation to the theme set out in the brief
- Facilitating access to appropriate resources where possible.
- Providing instructions at strategic intervals to facilitate the timely completion of the modelling project.
- Providing supports for students with special educational needs as outlined on page 16

Note that only work which is the student's own can be accepted for submission to the State Examinations Commission. It is not envisaged that the level of support involved requires teachers to edit draft reports, or to provide model text or answers to be used in the student's evidence of learning.

Inclusive practice and access arrangements

Leaving Certificate Applied Mathematics is designed to be accessible to every student. Any access arrangements that a school considers necessary for a particular student to carry out the course work component should be processed between the school and SEC as early as possible. These are known as reasonable accommodations. They are designed to enable the student to show what they know and what they can do without changing the demands of the assessment. It is important that, in order to make an informed decision before undertaking the course, any prospective learner who has a disability that might affect their capacity to engage with the standard assessment arrangements be made aware of the accommodations that are possible. Equally important is that the student be made aware, where relevant, of those access arrangements that are not possible. Further details as to the arrangements that are possible are available on the SEC's website, www.examinations.ie, or available from the Reasonable Accommodations Section of the SEC directly.

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Appendix 1:

Resource designed to support the development of the mathematical modelling competency

Note: This is NOT a sample modelling project, it is problem-based teaching module aligned with the specification and suitable for all abilities. [It can be accessed here.][5]

Problem scenario

Putting Data in the Saddle

Road cycling is a team sport, a sport of specialists playing their roles with split-second timing in support of an overall strategy for a stage, a tour, even a season. Climbers force the issue on hilly stages, exhausting competitors who try to keep touch; domestiques, junior riders, create a draft to minimize the effort of the star finishers (or you can use leader), dropping back to fetch water bottles and even giving up their bikes to senior riders with a flat or mechanical issue; all-rounders ("rouleurs", in French, the language of cycling) chase down opponents' breakaways, set up sprinters for the final mad dash, and serve as on-course captains when quick tactical decisions have to be made. Each specialty rewards different riding styles, different performance metrics, even different physiques. Collecting and analysing this data can help a team fit together the pieces of the strategic puzzle in a way that optimizes the use of team resources for a competitive edge.

Select one or more aspects of professional cycling. Use the mathematical modelling cycle to *formulate a problem, translate it to mathematics, find a solution to the problem, interpret the solution in relation to the original question* and *evaluate the model*

NTT Pro Cycling use big data, AI & machine learning to create smarter pro team
Bikerumour.com

"Most of the decisions by the team management, coaches and athletes were based on feelings and sensations... with all of the sophisticated sensors, technology, and knowledge, today's analytics is way more sophisticated than it used to be. For a cycling team, this is gold."

Pascal Hervé, Mentor of the team, retired French professional rider.

The only professional Canadian women's team with a license to race internationally, on the UCI Women's World Tour, is using state-of-the-art sensors and analytics software to fuel its rise though international (UCI) cycling ranks Sms.com

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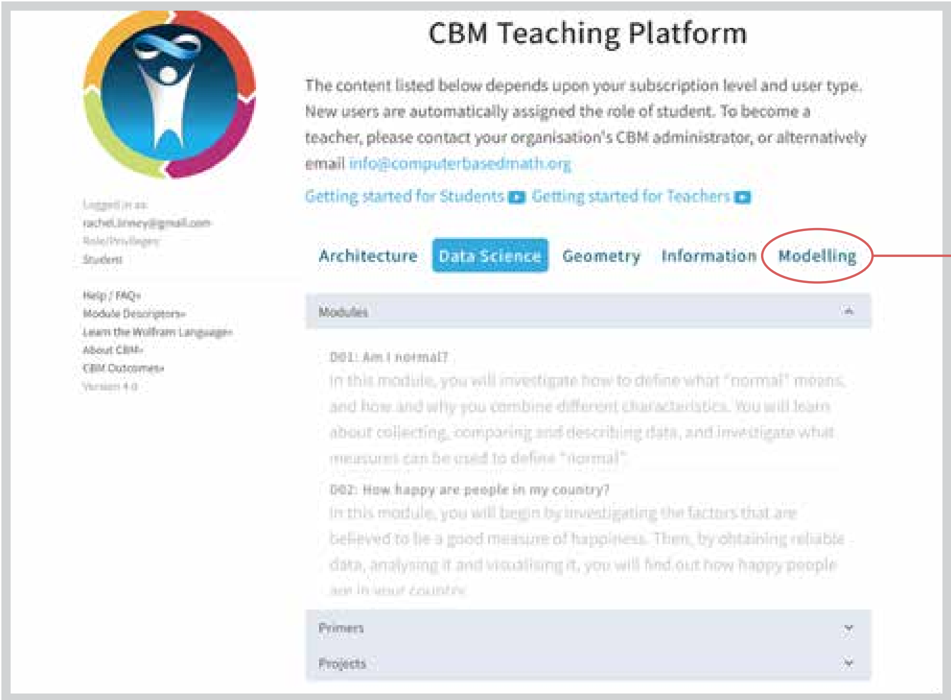
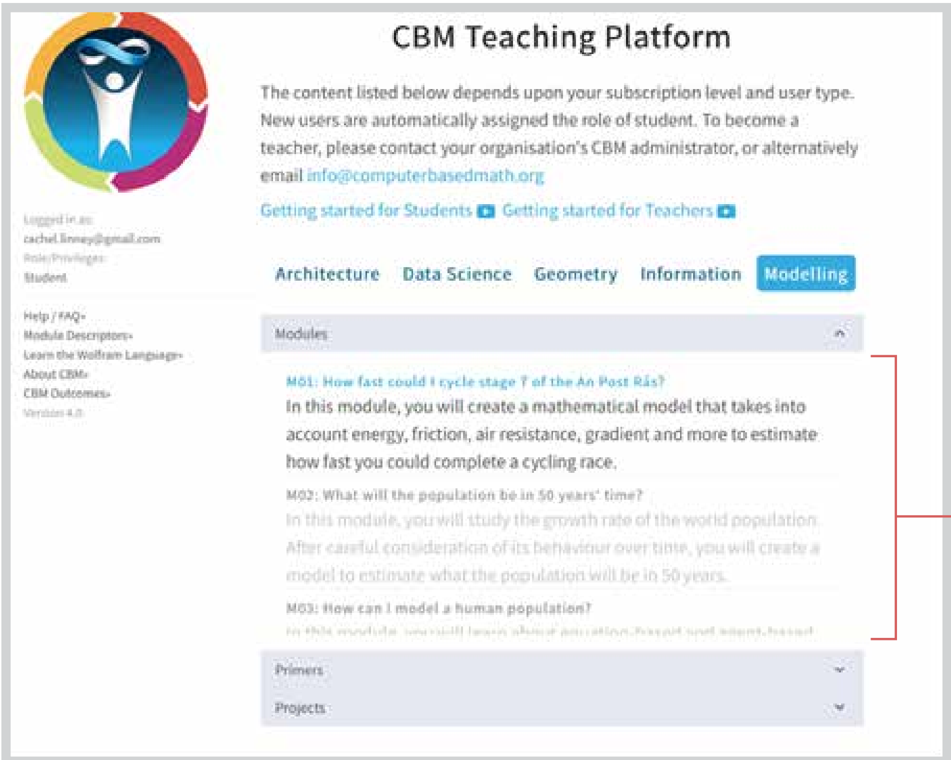
5 https://www.wolframcloud.com/env/CBM/TeachingPlatform/WelcomePage.nb

Landing Page



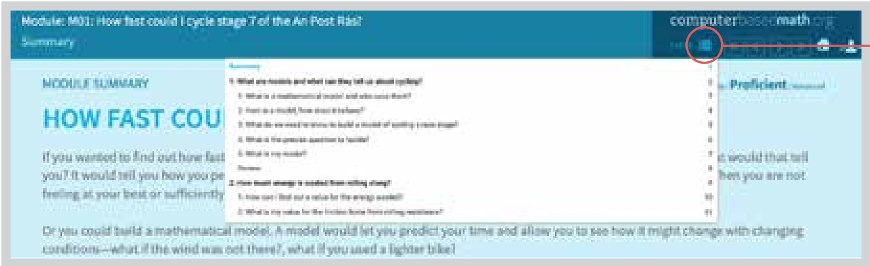
Click on Modelling



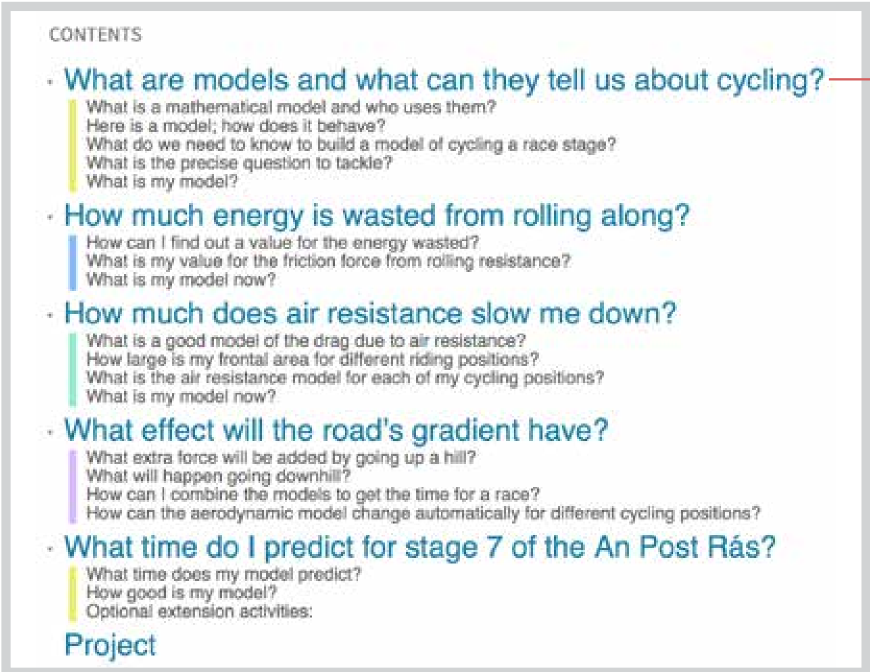
Module available here

Chapters and Activities

The supported learning is delivered within the context of a precise problem; How fast could I cycle stage 7 of the An Post Ras? The problem is derived from the problem scenario and the learning is split into chapters and activities, leading the learner through the mathematical modelling process with helpful checkpoints and opportunities to discuss progress at regular intervals.



Chapter Summary



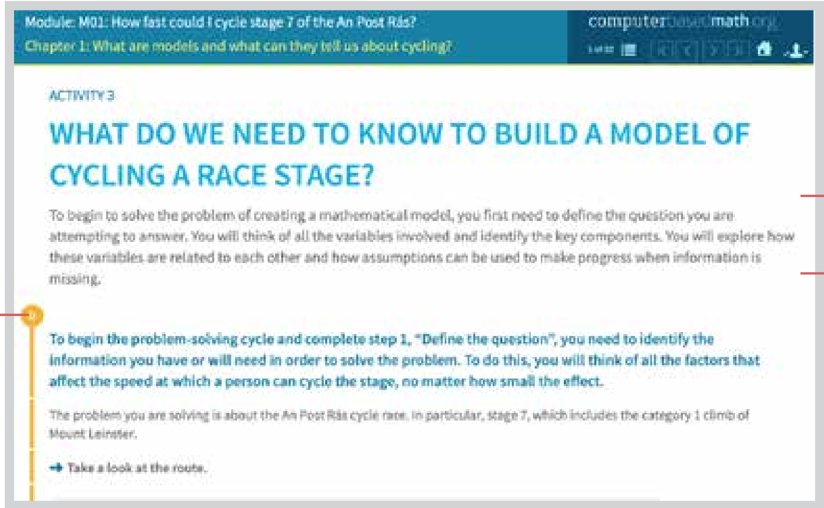
Each chapter title is a question that will be answered in that section, as is every activity title.

Learners experience in-context, real-life, messy problems and the activities required to make progress through the mathematical modelling process are explicitly signposted throughout to support the development of the mathematical modelling competency as a habit of mind.

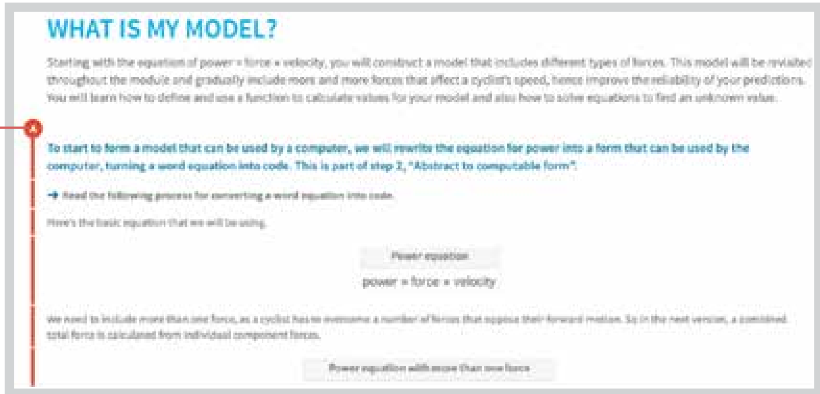
D	Formulating a problem: Defining the problem statement
A	Translating the problem to mathematics: Abstracting to mathematics
C	Computing and interpreting solutions: Computing a solution and understating the need to iterate
I	Evaluating and Reporting: Evaluating the model and interpreting the solution in the context of the original question.

Activity introduction

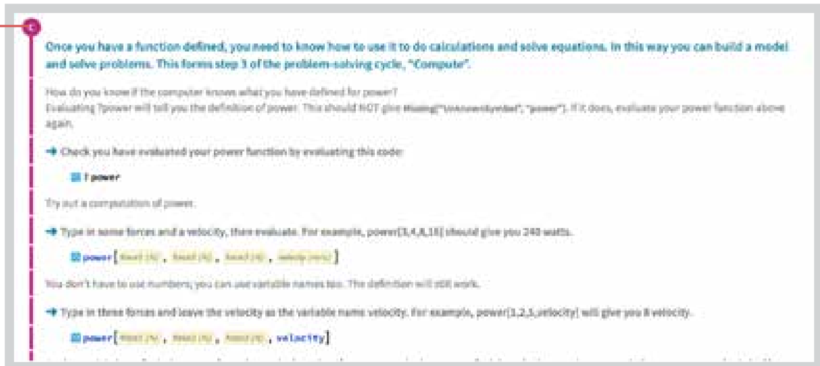
Here students are ***Formulating the problem*** *by defining the problem statement*



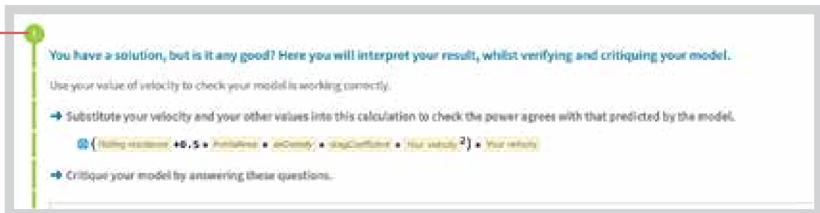
Here students are abstracting, ***Translating the problem to mathematics.***



Here students are using code to ***Compute and interpret*** *a solution to the problem.*

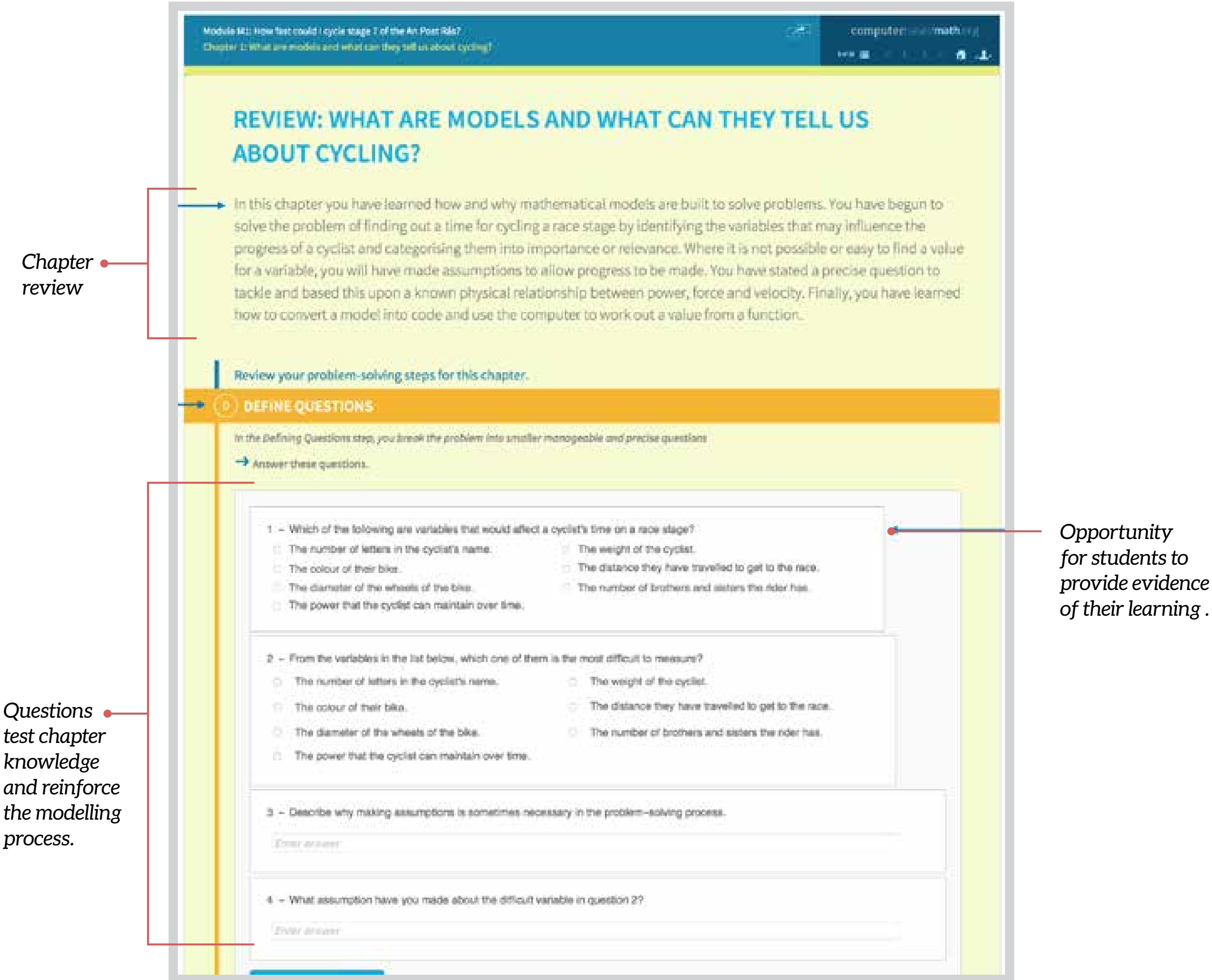


Here students are ***Evaluating and Reporting;*** *interpreting the solution in the context of the original problem and assessing the need to iterate by verifying and critiquing the model.*



Reflecting on the Learning

Reflection on learning is structured around the fundamental four-step modelling process, rather than the progressive steps of a calculation.



Assessing the Learning

A project provides opportunity for students to apply their learning by adapting their model to an unfamiliar context .

Guidance is given so as to embed the modelling process as a habit of mind.

NCCA
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

An Roinn Oideachais
Department of Education