



Oide

Tacú leis an bhFoghlaim
Ghairmiúil i measc Ceannairí
Scoile agus Múinteoirí

Supporting the Professional
Learning of School Leaders
and Teachers

Mathematics

2023 – 2024 Core Cluster



Oide

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Supporting the Professional
Learning of School Leaders
and Teachers

Welcome to the online cluster day for
Mathematics.

You are now logged in.

Before the day commences, you might like to
have a pen, a calculator and paper nearby. A
cup of tea or coffee might be nice also.

I'm looking forward to working with you today.

The first session will begin at 9:30am.



Padlet for today



@Oide_PPMaths



Introducing Oide



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Supporting the Professional
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Oide Mathematics Support



- Subject CPD Cluster Day
- Electives
- School Support
- Collaboratives
- Communications & Social Media
 - <https://oide.ie/post-primary>
 - @Oide_PPMaths 

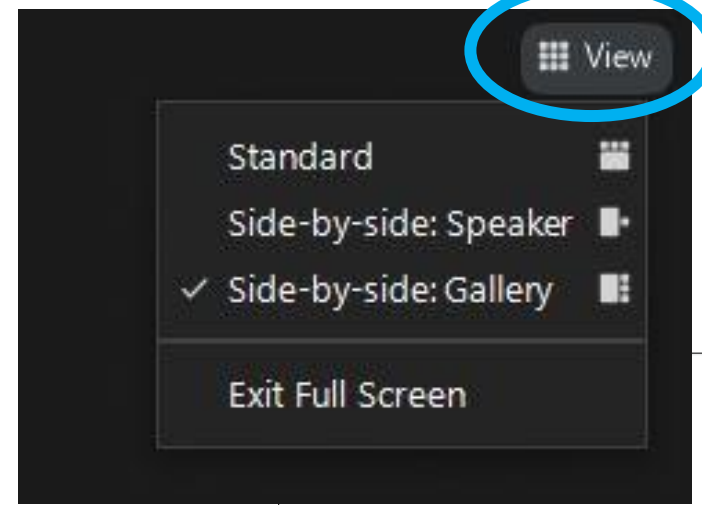


Zoom functions

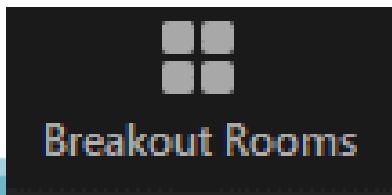
Speaker view

The tool bar

- Mute/Unmute microphones
- Turn cameras on/off
- Use the chat function to ask questions and input into discussions
- Share screen

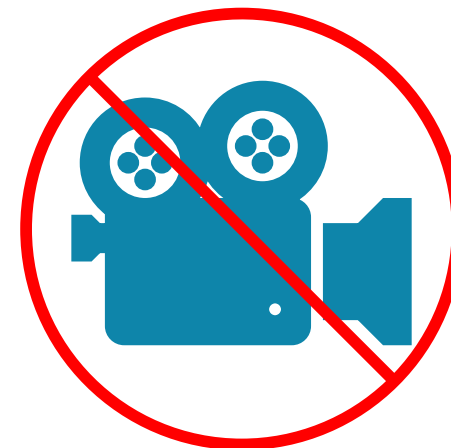
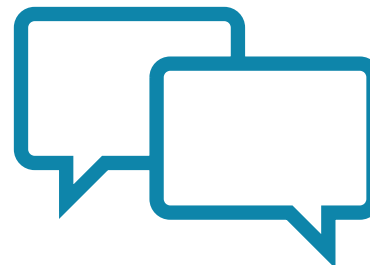


Breakout rooms





Before we begin...





Outline for the day

Approximate Time	Activity
09:15 – 09:30	Introduction
9:30 – 11:00	Exploring Mathematical Proficiency
11:00 – 11:15	Tea/Coffee
11:15 – 12:00	Exploring Mathematical Actions
12:00 – 13:00	Investigation in the Junior Cycle Mathematics Classroom
13:00 – 14:00	Lunch
14:00 – 15:30	Incorporating Rich Tasks into Classroom Practice



Teacher Professional Learning

Domain 4	Standard	Effective Practice	Highly Effective Practice
Teachers' Collective/ Collaborative Practice.	Teachers value and engage in professional learning and professional collaboration.	Teachers identify and engage in professional learning that develops their own practice and meets the needs of students and the school.	Teachers identify and engage in professional learning that develops their own practice, meets the needs of students and the school , and enhances collective practice.



School Context

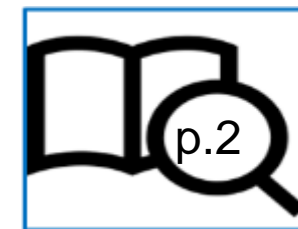
What is our context?

Who/what do we celebrate?

Student Context

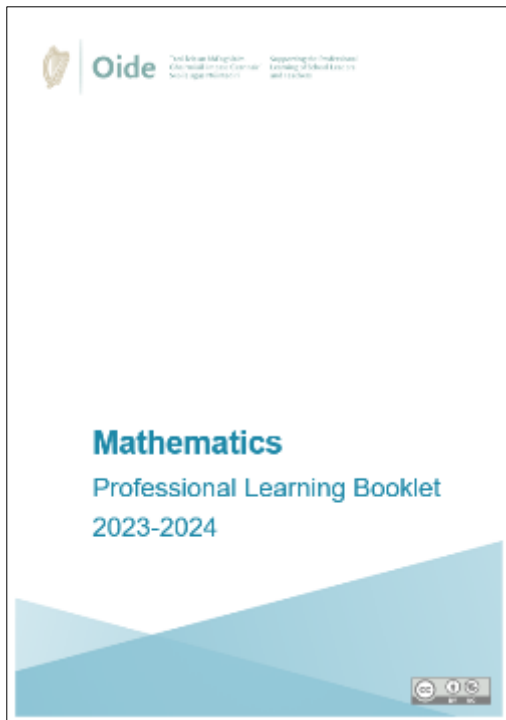
Who are my students?

Who am I planning for?





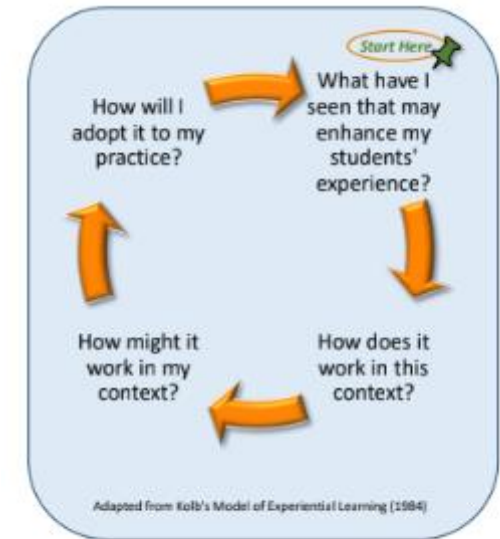
Resources for the day



Digital Booklet



Website Supports



Reflective Log



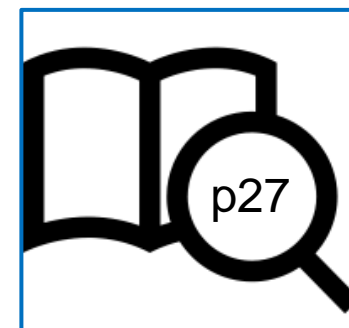
Reflective Log

What have I experienced that may enhance my students' experience?

How does it work in this context?

How might it work in my context?

How will I adapt/modify it for my practice?





Learning Intentions

- To identify and maximise opportunities for developing mathematical proficiency.
- To explore mathematical actions that support the development of mathematical proficiency.
- To discuss and reflect on how we support the incremental development of investigation skills for Junior Cycle Mathematics students.
- To further develop the incorporation of rich tasks into classroom practice.



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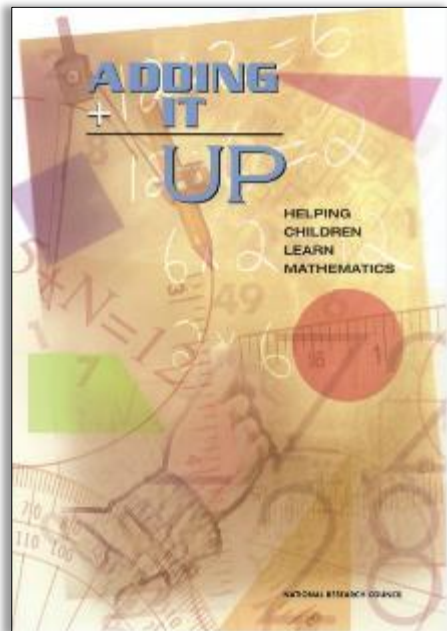
Supporting the Professional
Learning of School Leaders
and Teachers

Exploring Mathematical Proficiency



Learning Intention

To identify and maximise opportunities for developing mathematical proficiency.



‘The development of mathematical proficiency requires thoughtful planning, careful execution, and continual improvement of instruction. It depends critically on teachers who understand mathematics, **how students learn, and the classroom practices that support that learning.** They also need to know their students: who they are, what their backgrounds are, and what they know’

Adding it Up (2001) Kilpatrick et al



Learning Intention:

To identify and maximise opportunities for developing mathematical proficiency.

Success Criteria:

- I can describe how the components of mathematical proficiency are connected.
- I can explain the five components of mathematical proficiency.
- I can identify which components of mathematical proficiency are developed by various tasks.



Activity

Engage with three tasks in the context of a Junior Cycle Maths class

10 mins



Examine the components of mathematical proficiency



Discuss how components of mathematical proficiency can be developed using Tasks

15 mins





Planning Support for a Unit of Learning



An Roinn Oideachais
agus Scileanna

Junior Cycle

Unifying Strand

Number

Geometry &
trigonometry

Algebra &
functions

Statistics &
probability

Element: Building blocks

Element: Representation

Element: Connections

Element: Problem solving

Element: Generalisation and proof

Element: Communication

GT.3 investigate the concept of proof through their engagement with geometry so that they can:

- perform constructions 1 to 15 in *Geometry for Post-Primary School Mathematics* (**constructions 3 and 7 at HL only**)
- recall and use the concepts, axioms, theorems, corollaries and converses, specified in *Geometry for Post-Primary School Mathematics* (section 9 for OL **and section 10 for HL**)
 - axioms 1, 2, 3, 4 and 5
 - theorems 1, 2, 3, 4, 5, 6, 9, 10, 13, 14, 15 **and 11, 12, 19**, and appropriate converses, including relevant operations involving square roots
 - corollaries 3, 4 **and 1, 2, 5** and appropriate converses
- use **and explain** the terms: theorem, proof, axiom, corollary, converse, and implies
- create and evaluate proofs of geometrical propositions
- display understanding of the proofs of theorems 1, 2, 3, 4, 5, 6, 9, 10, 14, 15, **and 13, 19**; and of corollaries 3, 4, **and 1, 2, 5** (full formal proofs are not examinable)

Junior Cycle Mathematics Specification p.13



Learning Outcomes

GT3, GT4, U1, U2, U4, U5, U6, U7, U8, U9, U10, U11, U13

Context

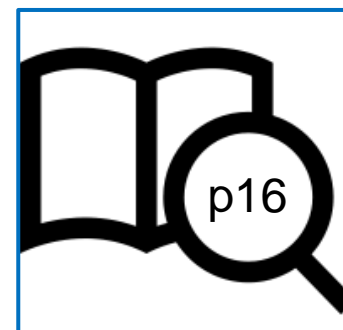
Consider your own context

Key Learning

- Students develop their mathematical arguments about geometric relationships
- Students apply Pythagoras' Theorem to solve right angle triangle problems
- Students analyse and interpret problems involving Pythagoras' Theorem
- Students develop their understanding of the construction of right-angled triangles

Ongoing Assessment

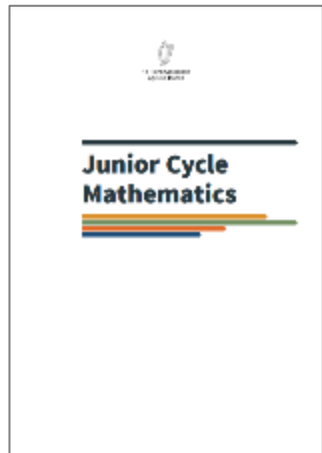
- Can students solve problems using Pythagoras Theorem in familiar and unfamiliar contexts?
- Can students develop a mathematical argument in relation to Pythagoras' Theorem?
- Can students select, justify and apply suitable constructions to solve problems involving right angled triangles?





Activity

Engage with three tasks in the context of a Junior Cycle Maths class



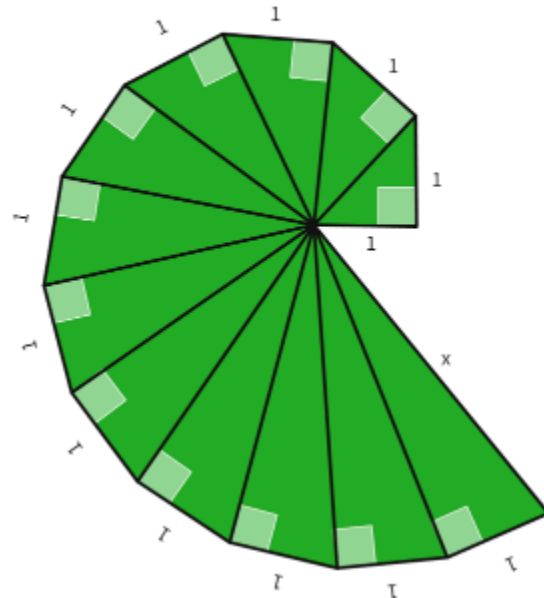
Right Angled Possibilities

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the length of the third side?

Can you explain your reasoning?

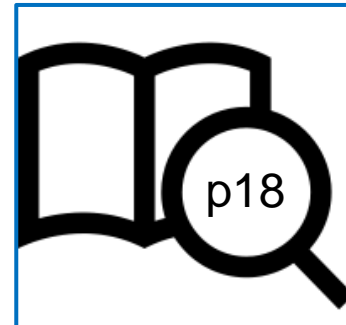
Pythagorean Shell

What missing side lengths can you find in this diagram?



Investigate what triangles can you form using side lengths from the squares (without overlapping squares)?

1cm ²	11cm ²	12cm ²
2cm ²		
3cm ²	10cm ²	13cm ²
4cm ²		
5cm ²	8cm ²	
6cm ²		
7cm ²	9cm ²	

10 mins



Activity

Engage with three tasks in the context of a Junior Cycle Maths class

10 mins



Examine the components of mathematical proficiency



Discuss how the components of mathematical proficiency can be developed using Tasks

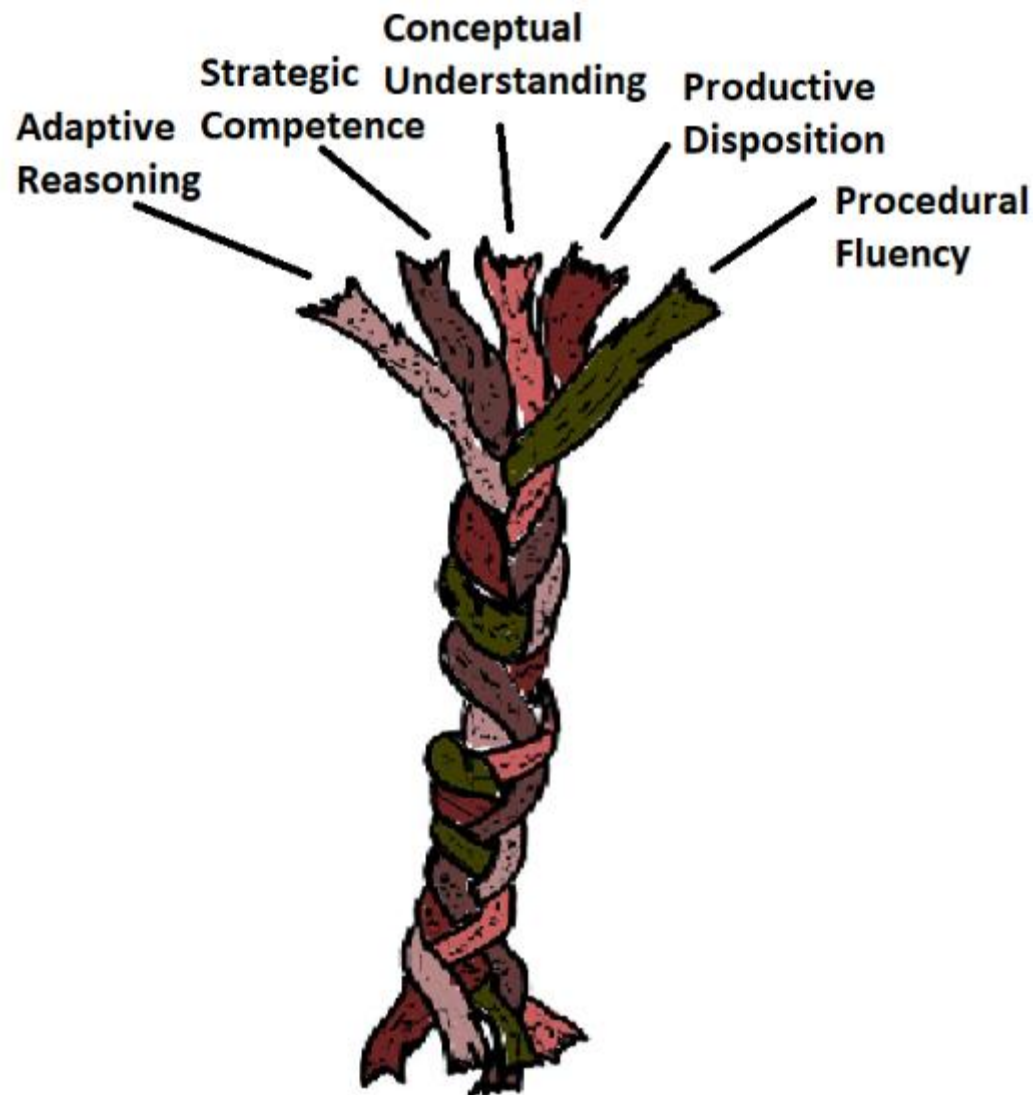
15 mins





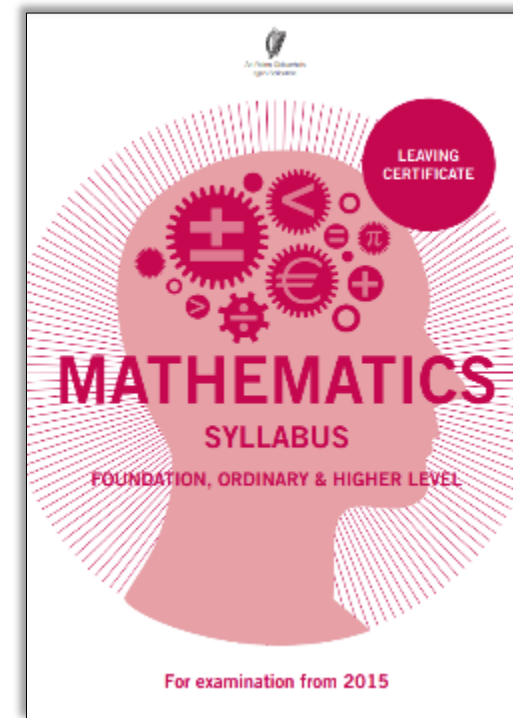
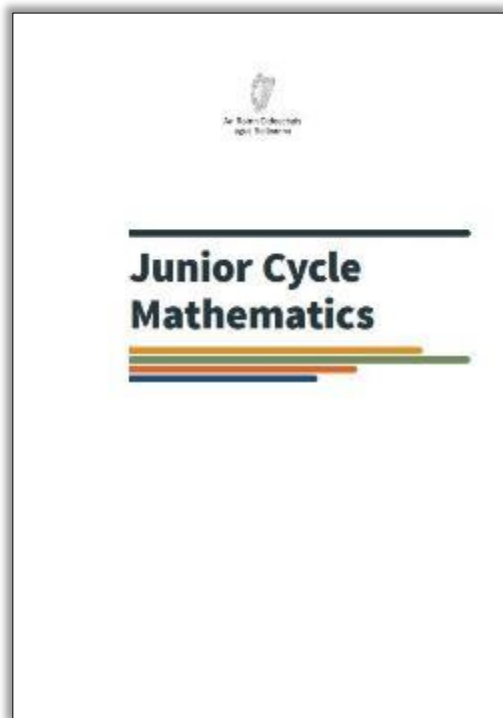
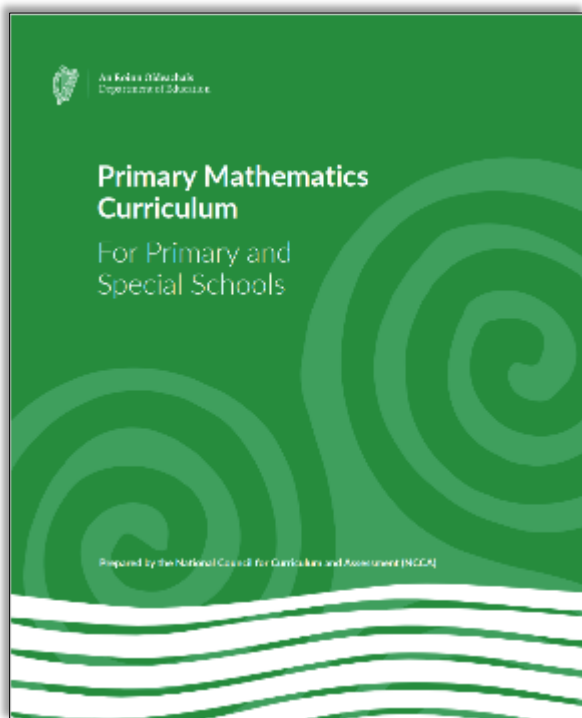
Activity

Examine the components of mathematical proficiency





Looking at the continuum

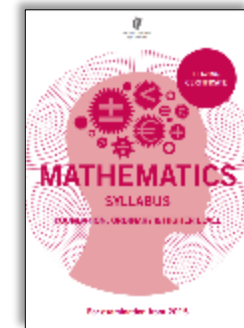
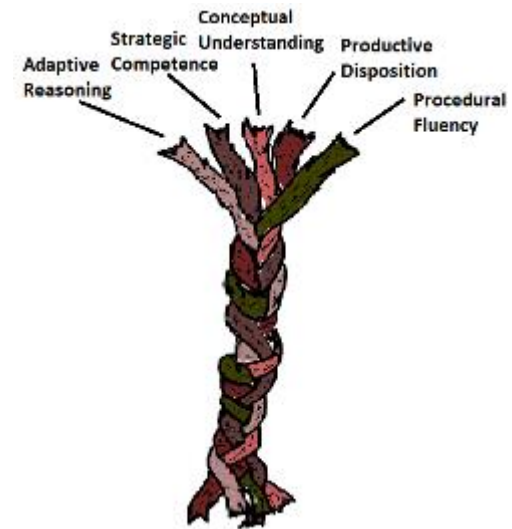
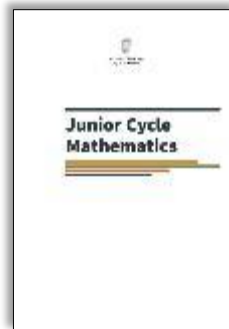




Mathematical Proficiency



Aim

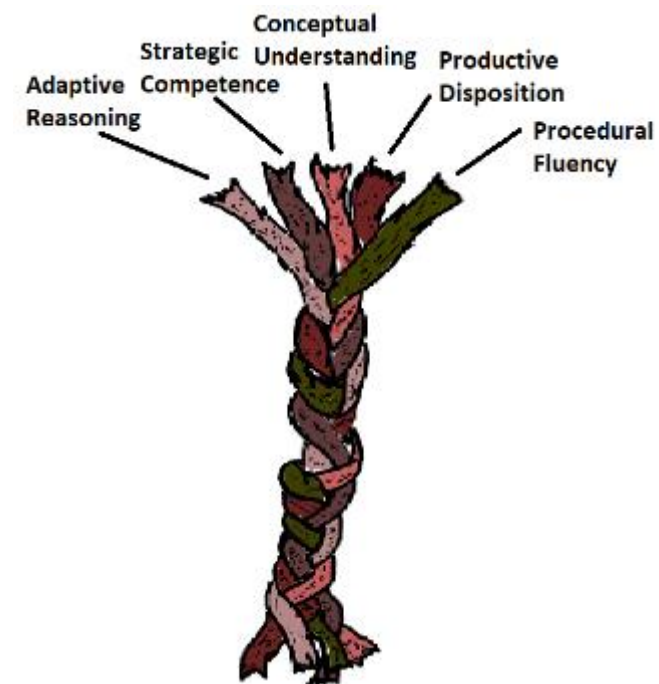


Objectives



Mathematical Proficiency

The mathematics that we are engaging in should be relevant for the students. It should be something that they are interested in, something relevant for their lives.



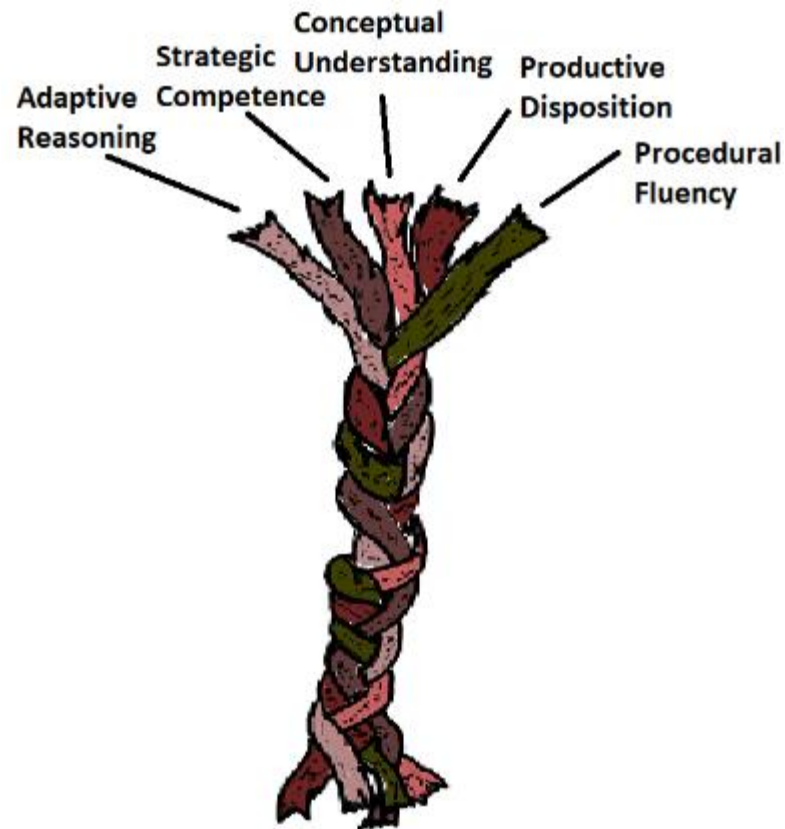
DCU Ollscoil Chathair
Bhaile Átha Cliath
Dublin City University

Sinead Breen





Mathematical Proficiency



‘...mathematical proficiency is conceptualised not as a one-dimensional trait but as having five interconnected and interwoven components.’

Junior Cycle Mathematics Specification p.5



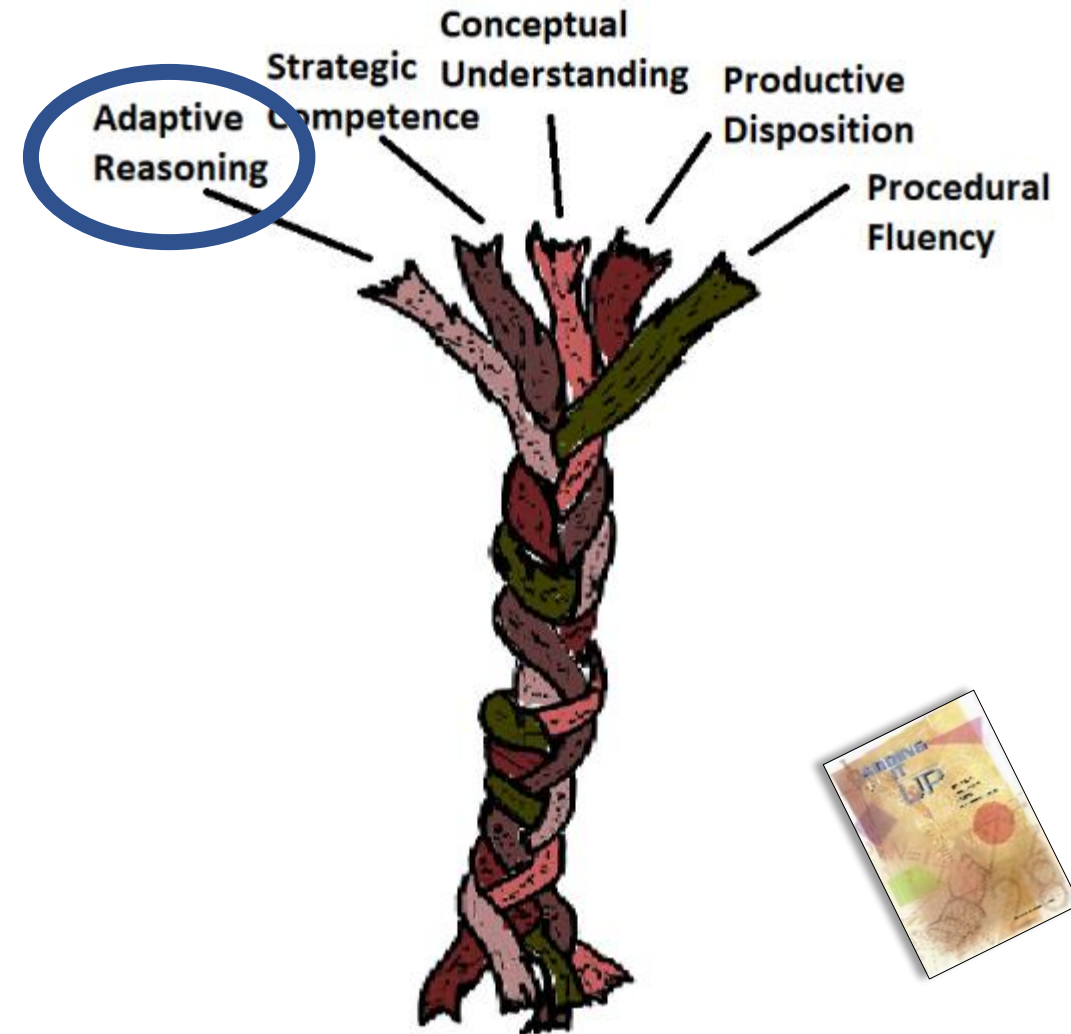
Adaptive Reasoning

‘The capacity to think logically about the relationships among concepts and situations-.

Junior Cycle Mathematics Specification p.5

Students with Adaptive Reasoning can

- justify and explain ideas in order to make their reasoning clear

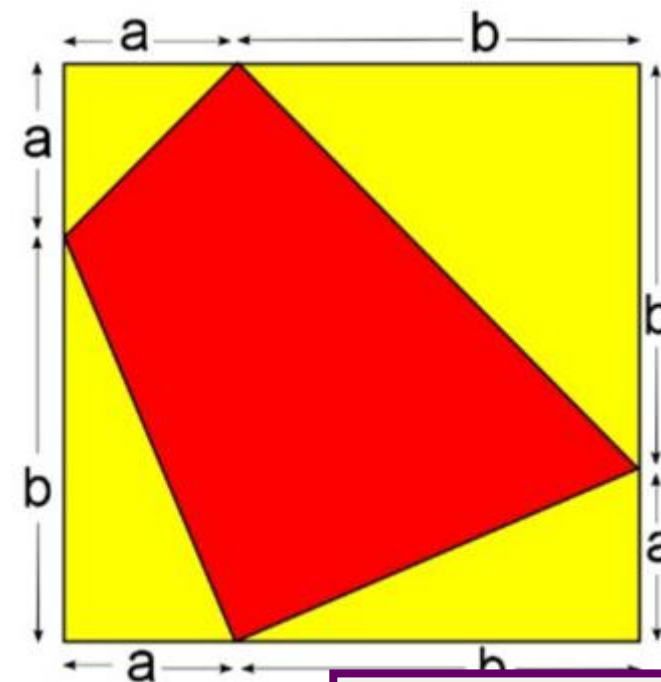
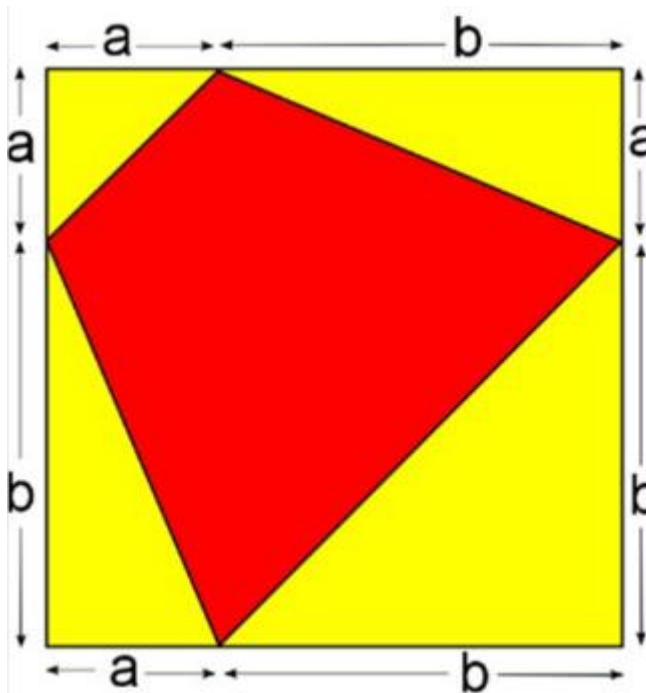




Adaptive Reasoning

Suppose we have a yellow square of side length $a+b$.

We can draw quadrilaterals in this square so that one vertex lies on each side of the square, and cuts each side into one segment of length a and one segment of length b , as below:



5 mins

Can you **prove** that in each of these images the area of the red quadrilateral is exactly half the area of the yellow square?



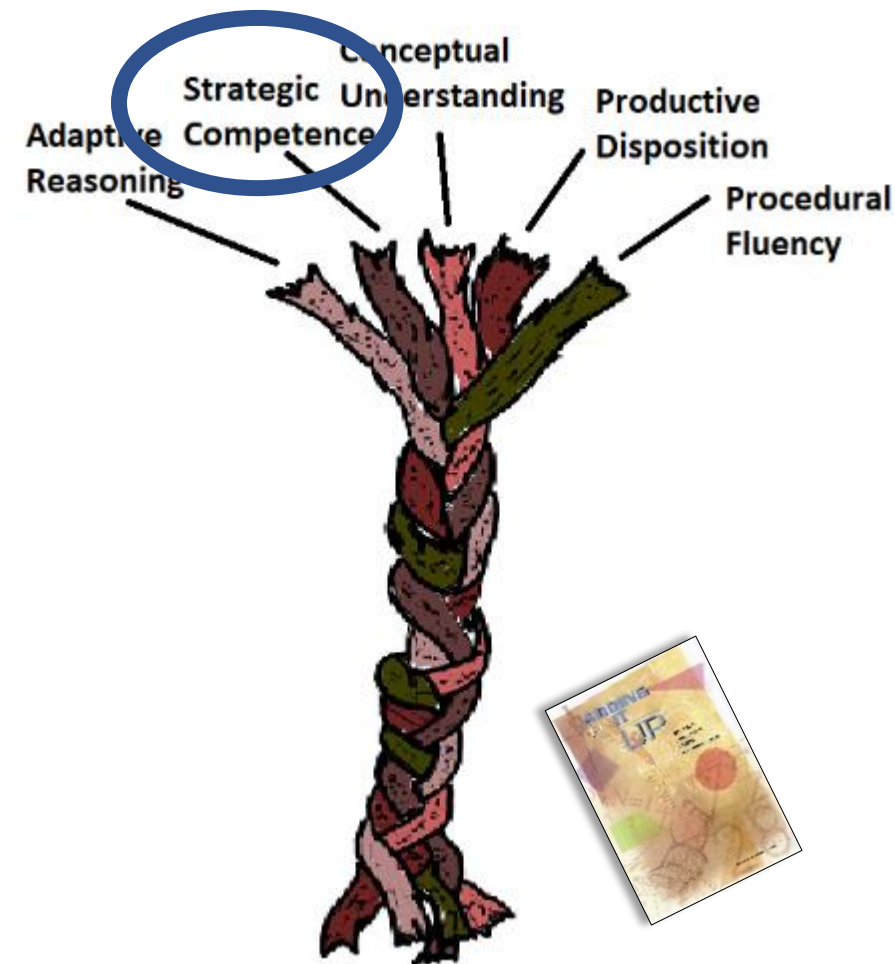
Strategic Competence

‘The ability to formulate, represent, and solve mathematical problems in both familiar and unfamiliar contexts’.

Junior Cycle Mathematics Specification p.5

Students with Strategic Competence can:

- formulate mathematical problems
- represent mathematical problems accurately
- develop possible solutions
- evaluate the accuracy of their solutions





Strategic Competence

Question Posing

Is there something that you would like to research or investigate based on the image?





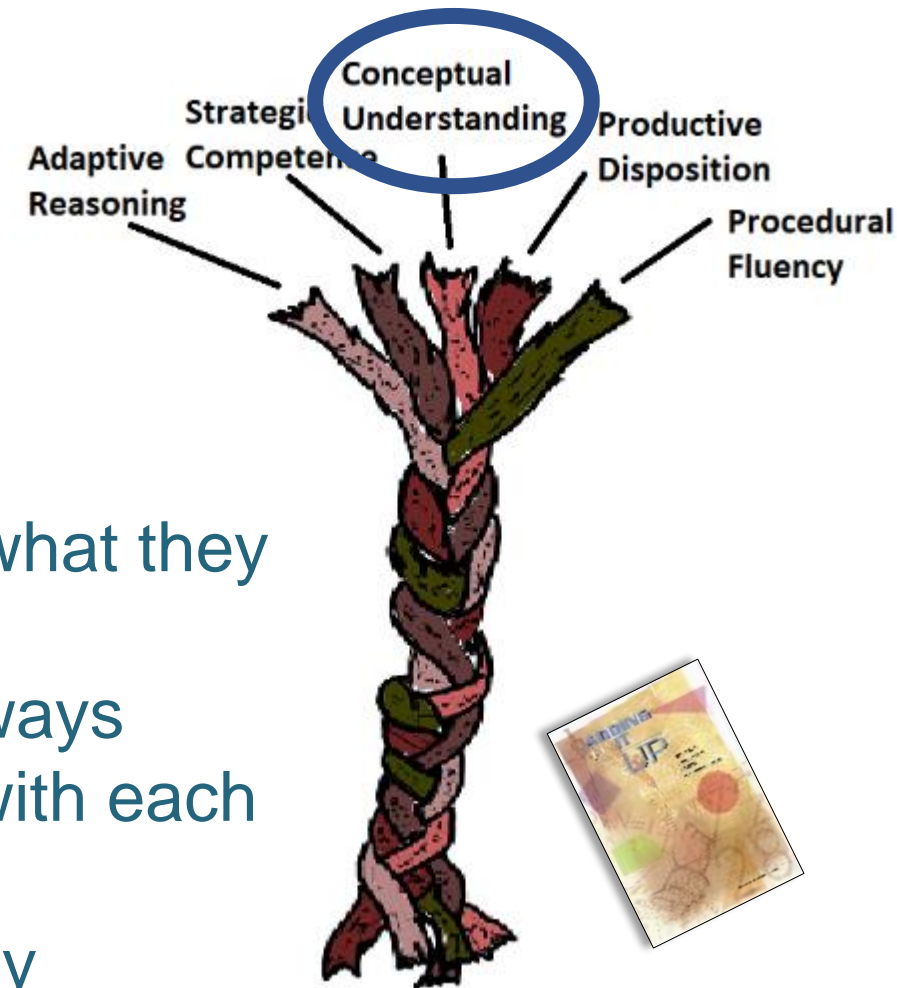
Conceptual Understanding

‘Comprehension of mathematical concepts, operations, and relations’

Junior Cycle Mathematics Specification p.5

Students with Conceptual Understanding can:

- learn new ideas by connecting those ideas to what they already know
- represent mathematical situations in different ways
- see how the various representations connect with each other
- see the deeper similarities between superficially unrelated situations

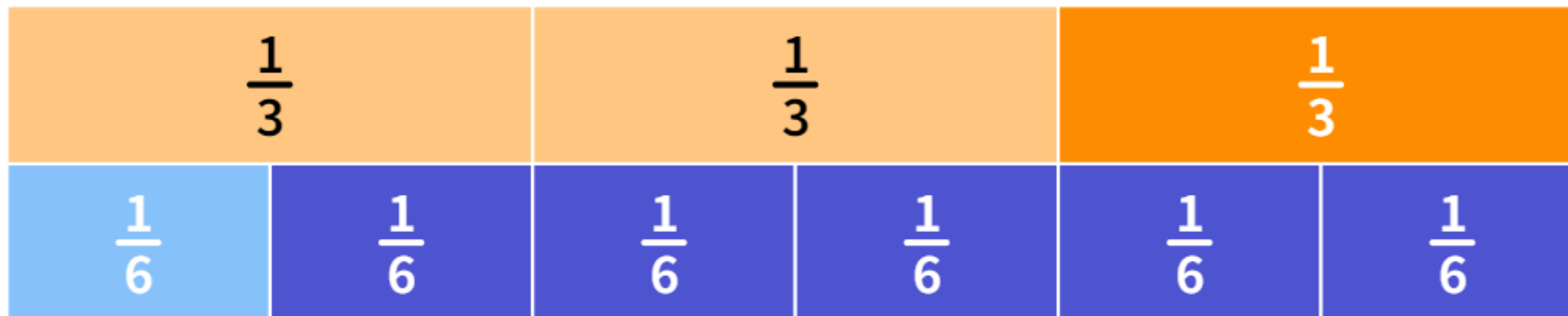




Conceptual Understanding

Representing concepts

$\frac{2}{3}$ divided by $\frac{1}{6}$





Productive Disposition

‘Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence, perseverance and one’s own efficacy’.

Junior Cycle Mathematics Specification p.5

Students with Productive Disposition believe:

- mathematics should make sense
- that they can solve mathematical problems through productive struggle
- becoming mathematically proficient is worth the effort.





Productive Disposition

Real Life Connections

<https://tinyurl.com/3mb5xyuh>

Brian Rushe
Dairy Farmer
Vice-President IFA





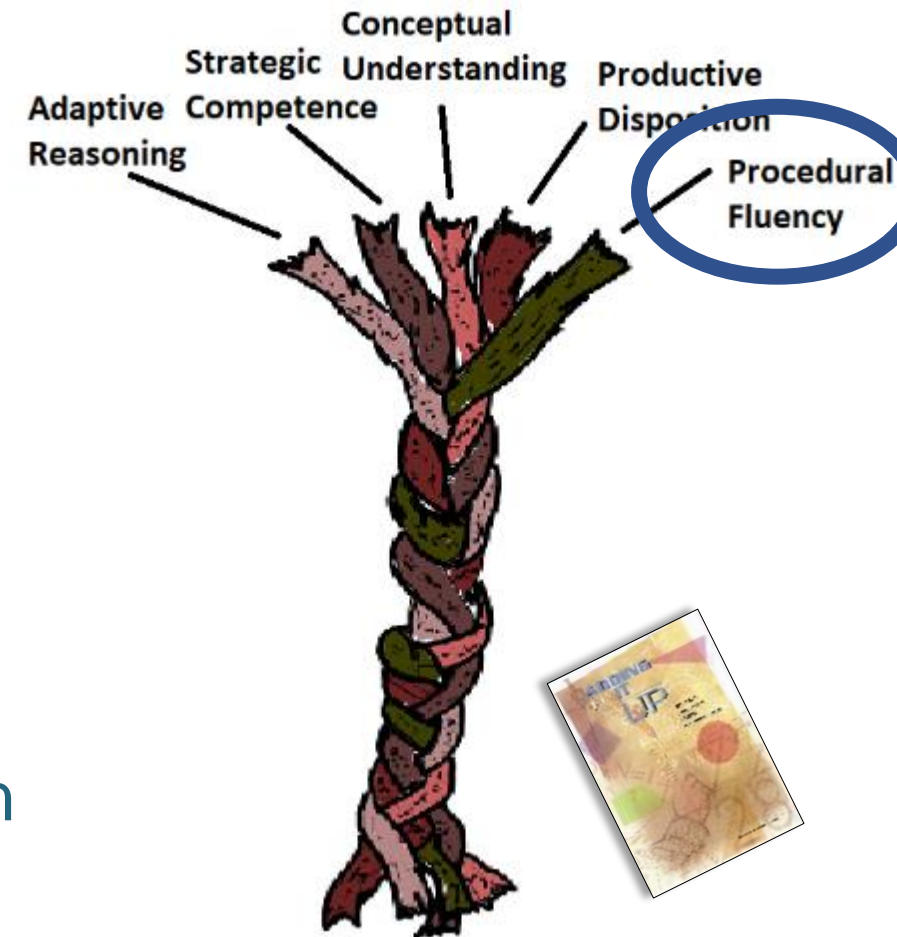
Procedural Fluency

Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately

Junior Cycle Mathematics Specification p.5

Students with Procedural Fluency can:

- estimate the result of a procedure
- practice of the skills they are learning
- apply procedures flexibly
- decide the appropriate tool for a given situation





Procedural Fluency

Practicing Procedures

Can you find every number between 1 and 20 using only 4's and any operation?



5 mins



Activity

Engage with three tasks in the context of a Junior Cycle Maths class

10 mins



Examine the components of mathematical proficiency



Discuss how components of mathematical proficiency can be developed using Tasks

15 mins



Components of Mathematical Proficiency

Conceptual Understanding
Strategic Competence
Productive Disposition
Procedural Fluency
Adaptive Reasoning
Adaptive Reasoning



Adaptive Reasoning

Students with Adaptive Reasoning can:

- justify and explain ideas in order to make their reasoning clear

Productive Disposition

Students with Productive Disposition believe:

- mathematics should make sense
- that they can solve mathematical problems through productive struggle
- becoming mathematically proficient is worth the effort

Conceptual Understanding

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- represent mathematical problems accurately
- develop possible solutions
- evaluate the accuracy of their solutions

'...mathematical proficiency is conceptualised not as a one-dimensional trait but as having five interconnected and interwoven components.'

Junior Cycle Mathematics Specification p.5

Strand	Geometry & Algebra	Algebra & Functions	Statistics & Probability
Problem Solving Skills	High	High	High
Number Operations	High	High	High
Problem Solving Skills	High	High	High
Number Operations	High	High	High
Problem Solving Skills	High	High	High
Number Operations	High	High	High





Activity

Discuss how components of mathematical proficiency may be developed within a lesson(s) using the three tasks



Discuss

1. Which component(s) could be developed by engaging with each of these tasks?

Discuss

2. Can you think how these tasks could be adapted to emphasise an alternative component?



15 mins



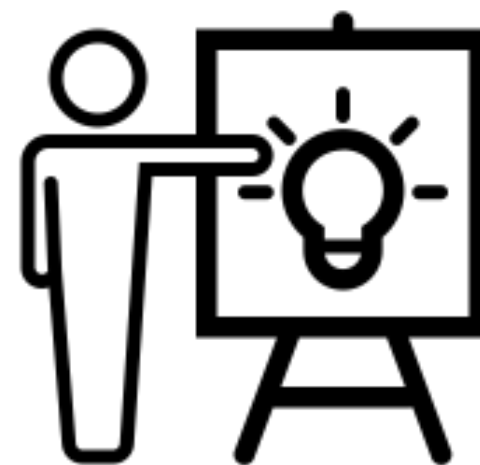
Share our thinking

Discuss

1. Which component(s) could be developed by engaging with each of these tasks?

Discuss

2. Can you think how these tasks could be adapted to emphasise an alternative component?





Learning Intention:

To identify and maximise opportunities for developing mathematical proficiency.

Success Criteria:

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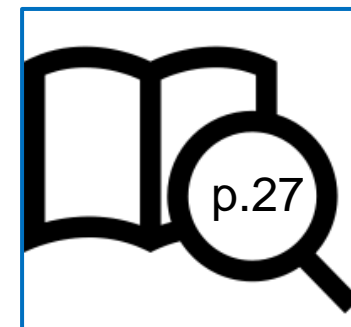
Reflective Log

What have I experienced that may enhance my students' experience?

How might it work in my context?

How does it work in this context?

How will I adapt/modify it for my practice?



3 mins



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Coffee Break



15 mins





Outline for the day

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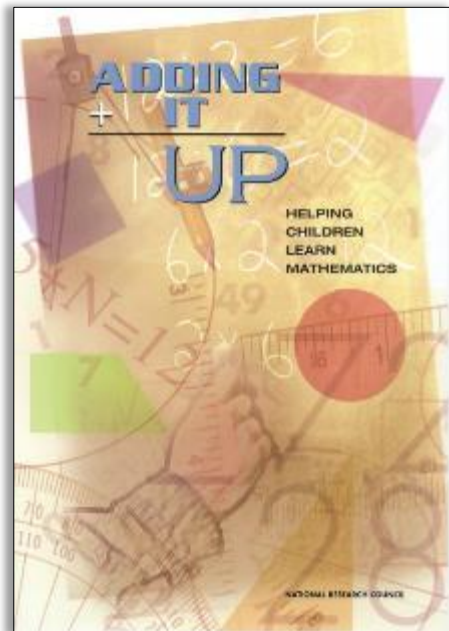
Supporting the Professional
Learning of School Leaders
and Teachers

Exploring Mathematical Actions



Learning Intention

To explore **mathematical actions** that support the development of mathematical proficiency.



‘To develop mathematical proficiency, we believe that students require more than just the demonstration of procedures. **They need experience in investigating** mathematical properties, **justifying** solution methods, and **analysing** problem situations.’

Adding it Up (2001) Kilpatrick et al



Learning Intention:

To explore **mathematical actions** that support the development of mathematical proficiency.

Success Criteria:

- I can define and use Junior Cycle Mathematics action verbs.
- I can identify appropriate actions to support the development of my students' mathematical proficiency.
- I can write student-centred success criteria, using the action verbs.



Student-Centred Learning

I think unless we establish the environments where we realise mistakes are actually going to be beneficial,

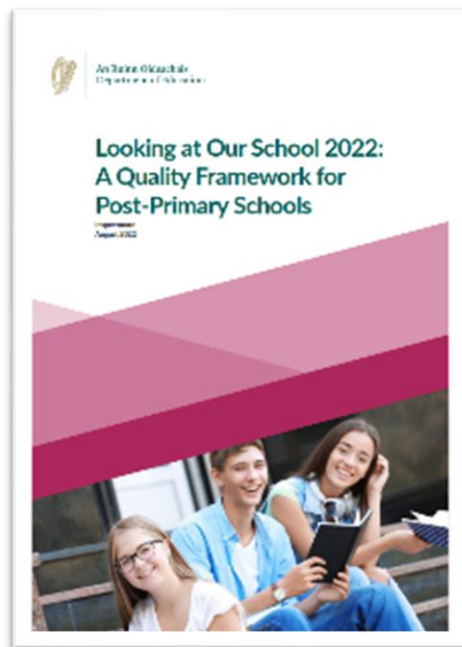


ASSOC PROFESSOR
Aoibhinn Ni
Shuilleabhain

‘We need to focus on what is the work that our students are doing...’



Student-Centred Learning



“Students as active agents in their learning:

The quality framework views students as active agents in their learning who **engage purposefully in a wide range of learning activities**, who respond in a variety of ways to different learning opportunities and who are afforded the opportunity to engage in meaningful discussions with teachers to inform learning and teaching.”

Looking at Our School 2022



Activity

Consider Action Verbs

10 mins



Select a task and write Success Criteria using Action Verbs

20 mins



Share the Success Criteria for each task





Activity

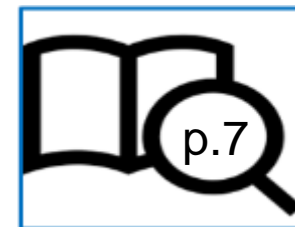


Take a moment to consider the Junior Cycle Mathematics Action Verbs



Discuss:

Reflect on the meaning of the Action Verbs in the context of the Junior Cycle Mathematics Learning Outcomes



10 mins



What are Success Criteria?



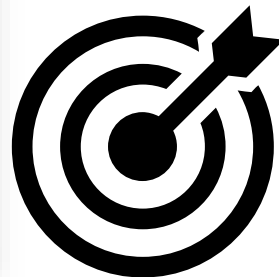
“Success Criteria are linked to learning intentions.
Success Criteria are developed by the teacher and/or the student
Success Criteria describe what success looks like.

(NCCA, Focus on Learning p.5)





Success Criteria



‘They help the teacher and student to make judgements about the quality of student learning’.

‘Research shows that students who regularly receive Success Criteria are:

- More focused for longer periods of time
- More motivated and active in the learning
- Better equipped to take ownership of their learning’

(NCCA, Focus on Learning p.5, 22)



Success Criteria

Domain 3	Standard	Effective Practice	Highly Effective Practice
Teachers' Individual Practice.	The teacher selects and uses planning, preparation and assessment practices that progress students' learning	Teachers share success criteria with students so that they can assess their own learning through self-assessment and peer assessment.	Teachers share and co-create success criteria with students so that they can assess their own learning through self-assessment and peer assessment, and identify areas for improvement and strategies to achieve improvement



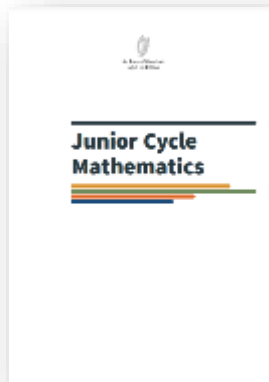
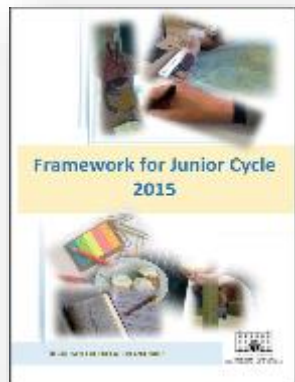


Managing Myself



‘This key skill helps learners to understand themselves both as individuals and as learners so that they can develop personal goals and plans. It also helps them develop strategies to make considered decisions, to take action and to reflect on their progress’.

NCCA Key Skills of Junior Cycle p.4





Activity

Consider Action Verbs

10 mins



Select a task and write Success Criteria using Action Verbs

20 mins



Share the Success Criteria for each task





Writing Success Criteria

Sample Learning Intention

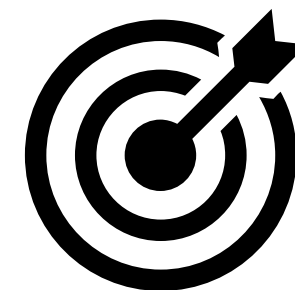
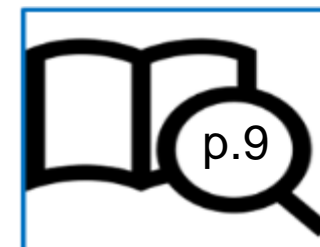
We are learning about the properties of triangles

Sample Success Criteria

- 🎯 I can **sketch** right angled triangles
- 🎯 I can **solve** problems involving Pythagoras' Theorem
- 🎯 I can **explain** my reasoning

Right Angled Possibilities:

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the third side?





Activity

Writing Success Criteria using Action Verbs

Using tasks provided and keeping the action verbs in mind, write Success Criteria for one of these tasks



Classroom Context

- Mixed ability
- 24 students
- 2 exceptionally able students
- 1 student with EAL
- 2 students have dyslexia



Activity

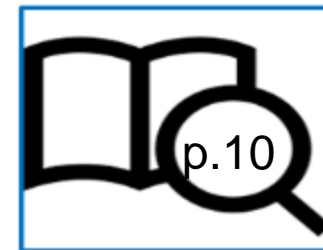
Writing Success Criteria using Action Verbs

Using tasks provided and keeping the action verbs in mind, write Success Criteria for one of these tasks



Examine the tasks

1. Identify the actions needed to engage with the tasks
2. Write Success Criteria for one of these tasks
3. Can you add an extension to this task to create additional challenge?



20 mins



Activity

Consider Action Verbs

10 mins



Select a task and write Success Criteria using Action Verbs

20 mins



Share the Success Criteria for each task





Share our thinking

Share the Success Criteria for each task



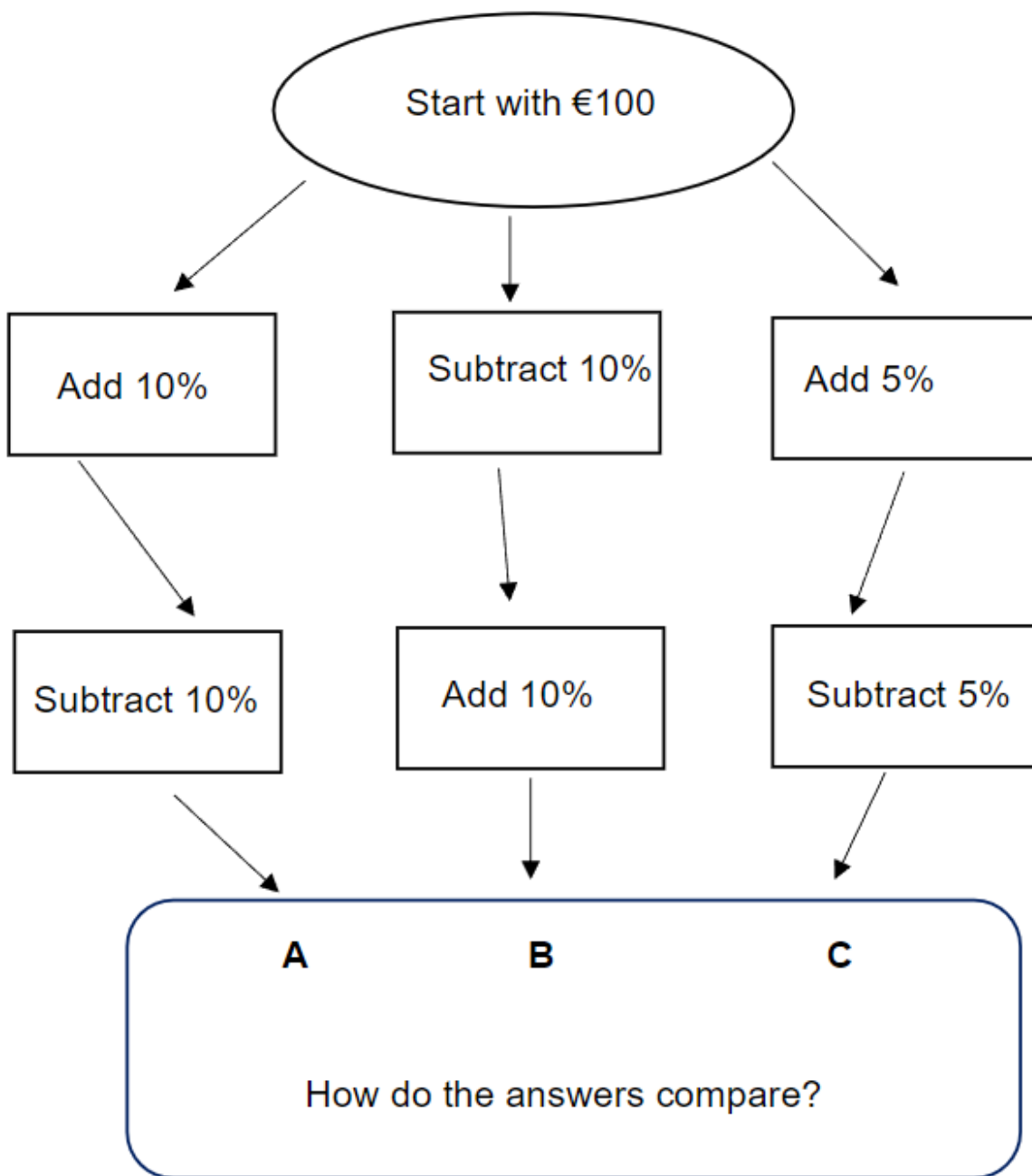
Discuss:

1. Identify the actions needed to engage with the tasks
2. Write Success Criteria for one of these tasks
3. Can you add an extension to this task to create additional challenge?



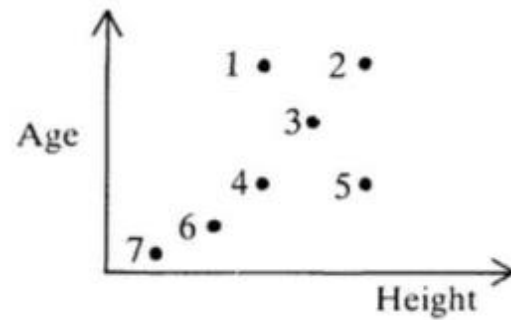
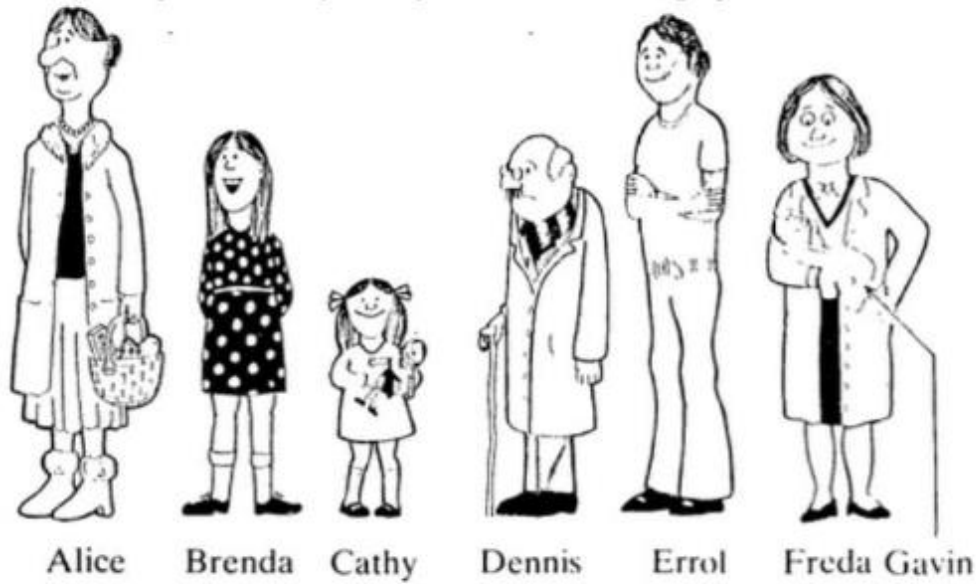
Examine the image below. Can you fill in the boxes to indicate the general term for the following shaded sequences. The first one is already completed.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	$4n$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	$5n$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	$6n - 5$



Who is represented by each point on the scatter graph below?

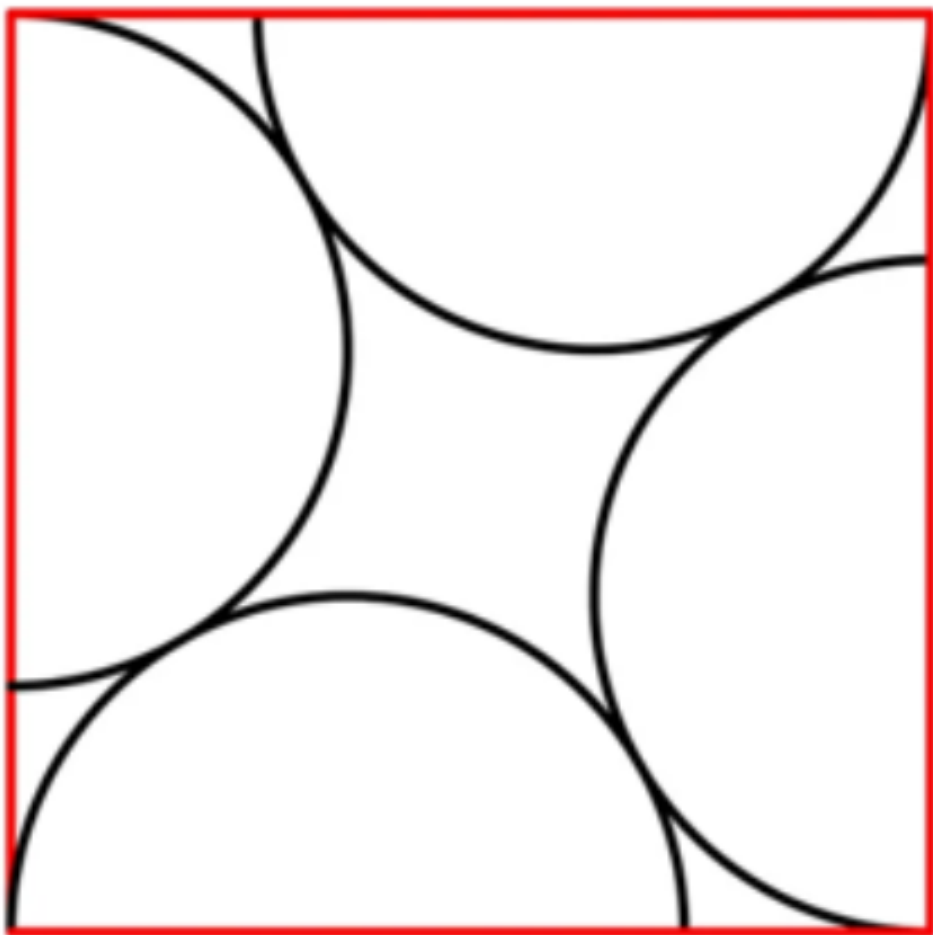
Justify your reasoning.





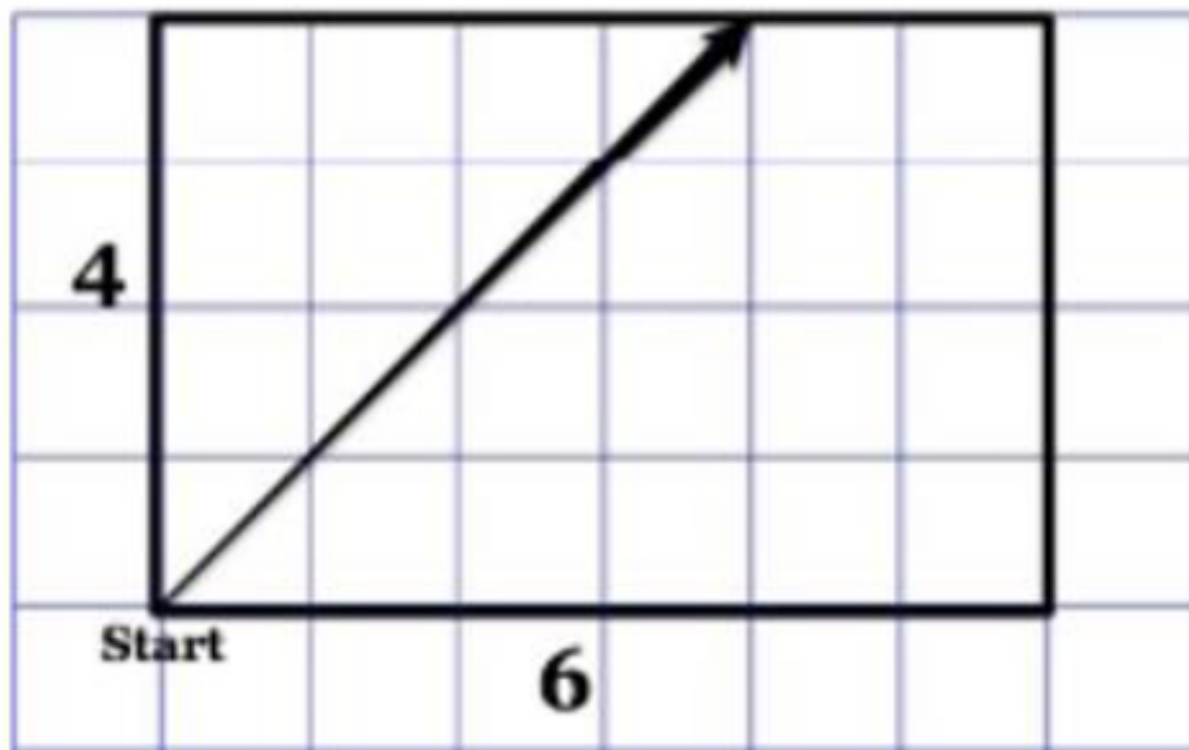
Four semicircles with radius 2 are constructed in the red square below.

What is the area of the square?

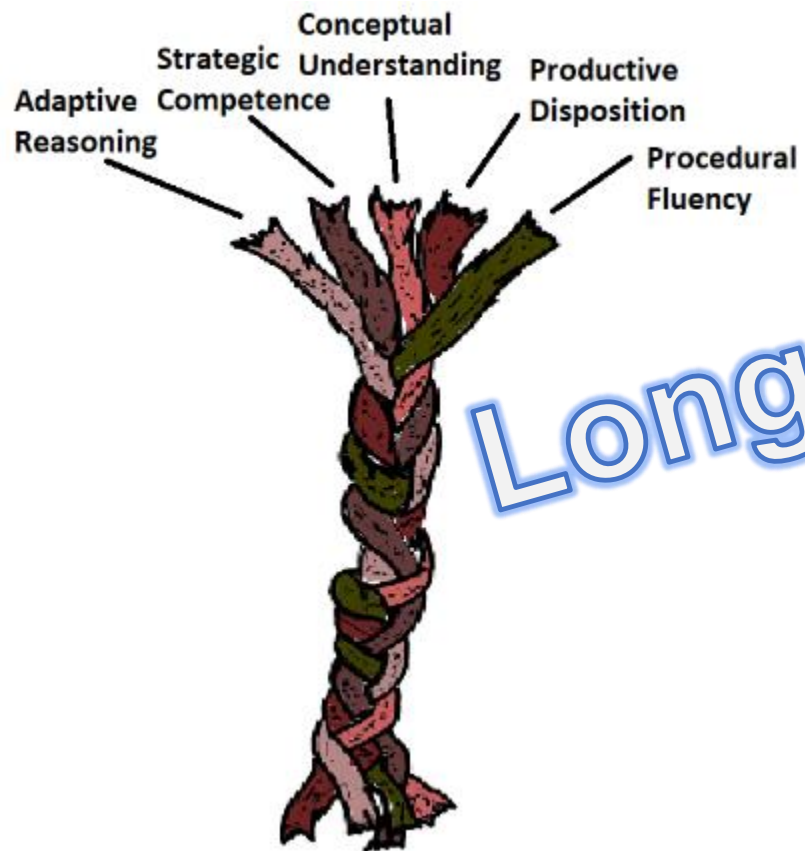




Start with the 4 x 6 pool table below with a pocket in each corner. The ball always launches from the bottom left corner of a rectangular table at 45 degrees, and bounces until it reaches a corner. If the ball continues to roll which pocket will the ball drop into?



Now consider tables of different dimensions. Which pocket will the ball drop into?



Aim for our students

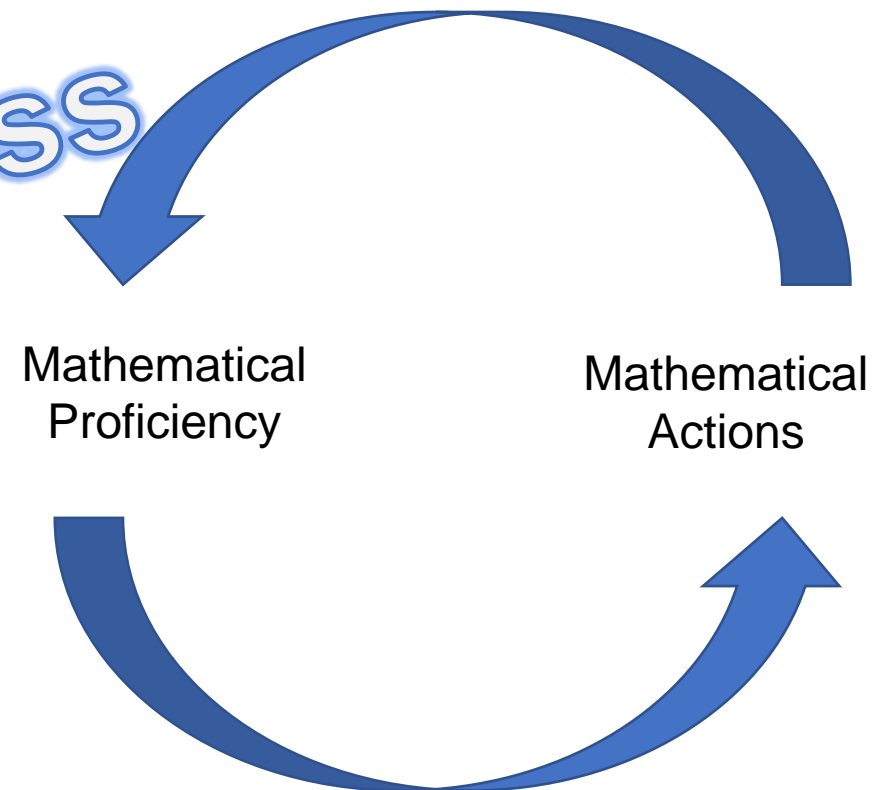
25 Junior Cycle Mathematics Appendix A: Glossary of action verbs

This glossary is designed to clarify the learning objectives and to help in the selection of what the learner should be able to do or create. The glossary will be aligned with the command verbs used in the curriculum.

Action verb	Definition
Analyse	break down into parts to bring out the essential parts and relationships, and to identify patterns or relationships
Apply	use knowledge and skills to solve a problem in a new situation
Calculate	work out a numerical answer
Classify	group things based on common characteristics
Compare	give an account of the similarities and/or differences between two (or more) items or situations, referring to both (all) of them throughout
Construct	use properties of shapes and geometric results to draw accurately, using only the prescribed geometrical tools
Convert	change from one form to another
Define	state a rule that identifies the elements of a set
Discuss	offer a considered, balanced review that includes a range of arguments, factors or hypotheses; opinions or conclusions should be presented clearly and supported by appropriate evidence
Estimate	state or calculate a rough value for a particular quantity
Evaluate	judge the relative quality or validity of something, which may include analysing, comparing and contrasting, criticising, defending, or judging
Explain	give a reasoned account, showing how causes lead to outcomes
Generalise	generate a general statement based on specific instances
Generate	produce or create
Interpret	use knowledge and understanding to explain the meaning of something in context
Investigate	observe, study, or make a detailed and systematic examination to establish facts and reach new conclusions
Justify	give valid reasons or evidence to support an answer or conclusion
Mathematise	generate a mathematical representation (e.g. graph, equation, geometric figure) to describe a particular aspect of a phenomenon

Process

Long term process



Interdependent



Learning Intention:

To explore **mathematical actions** that support the development of mathematical proficiency.

Success Criteria:

- I can define and use Junior Cycle Mathematics action verbs.
- I can identify appropriate actions to support the development of my students' mathematical proficiency.
- I can write success criteria, using the action verbs, given a particular context.



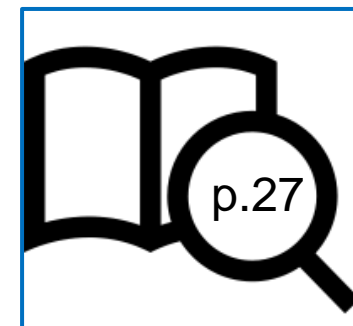
Reflective Log

What have I experienced that may enhance my students' experience?

How might it work in my context?

How does it work in this context?

How will I adapt/modify it for my practice?



3 mins



Oide

Tacú leis an bhFoghlaim
Ghairmiúil i measc Ceannairí
Scoile agus Múinteoirí

Supporting the Professional
Learning of School Leaders
and Teachers

Investigation in the Junior Cycle Mathematics Classroom



Outline for the day

Approximate Time	Activity
09:15 – 09:30	Introduction
9:30 – 11:00	Exploring Mathematical Proficiency
11:00 – 11:15	Tea/Coffee
11:15 – 12:00	Exploring Mathematical Actions
12:00 – 13:00	Investigation in the Junior Cycle Mathematics Classroom
13:00 – 14:00	Lunch
14:00 – 15:30	Incorporating Rich Tasks into Classroom Practice



Learning Intention:

To discuss and reflect on how we support the incremental development of investigation skills for Junior Cycle Mathematics students.

Success Criteria:

- I can explain the elements of investigative tasks.
- I can identify occasions where investigative tasks can benefit development of my student's mathematical proficiency.



Learning Intention

To discuss and reflect on how we support the incremental development of investigation skills for Junior Cycle Mathematics students.

What is our current understanding of investigation for mathematics students?



'Natural numbers can be expressed as the sum of two or more consecutive positive whole numbers'

**Investigate this statement.
What conclusions can be drawn?
Justify your reasoning.**

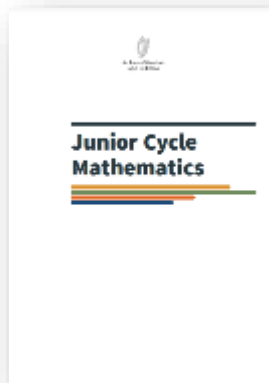
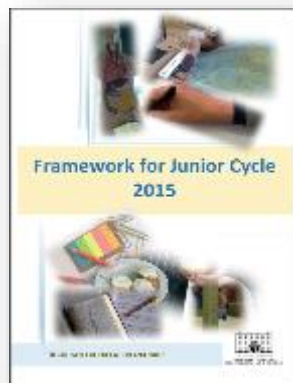


Managing Information and Thinking



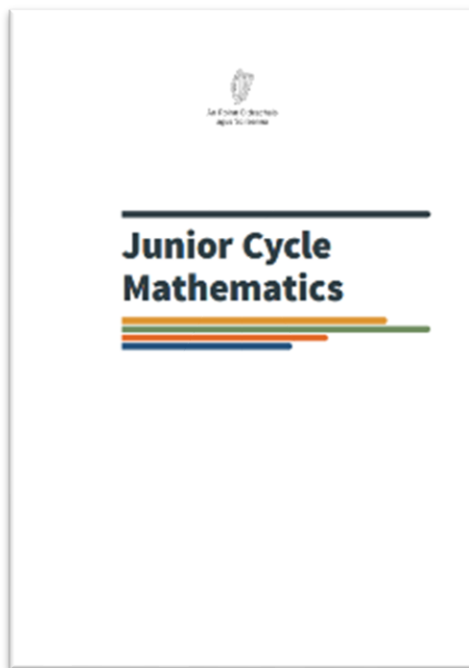
‘This skill also develops learners’ thinking skills so that they can become more skilled in higher order reasoning and problem-solving’.

NCCA Key Skills of Junior Cycle p.4





What does it mean to investigate?



Investigate - Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions

Junior Cycle Maths Specification p.25



Investigative Approaches

“In order to make an introduction to a topic more open you can alter your approach so that students may:

- Make decisions en route;
- Construct their own examples;
- Draw their own conclusions.”

(Onion et. al) Investigative and Problem-Solving Approaches to Mathematics and their Assessment





Activity

Engage with an investigative task

15 mins



Hear from a teacher and observe students

10 mins



Discussion on investigative tasks

10 mins





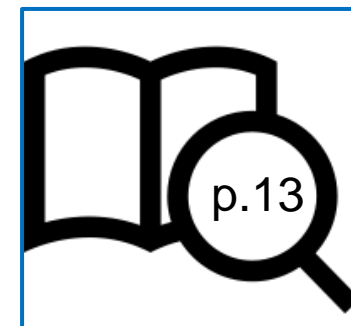
Investigative Activity

Engage with an investigative task



While engaging with the following tasks discuss:

- What are the skills are developed by an investigative task?
- Which Components of Mathematical Proficiency can be developed by investigative work?



15 mins

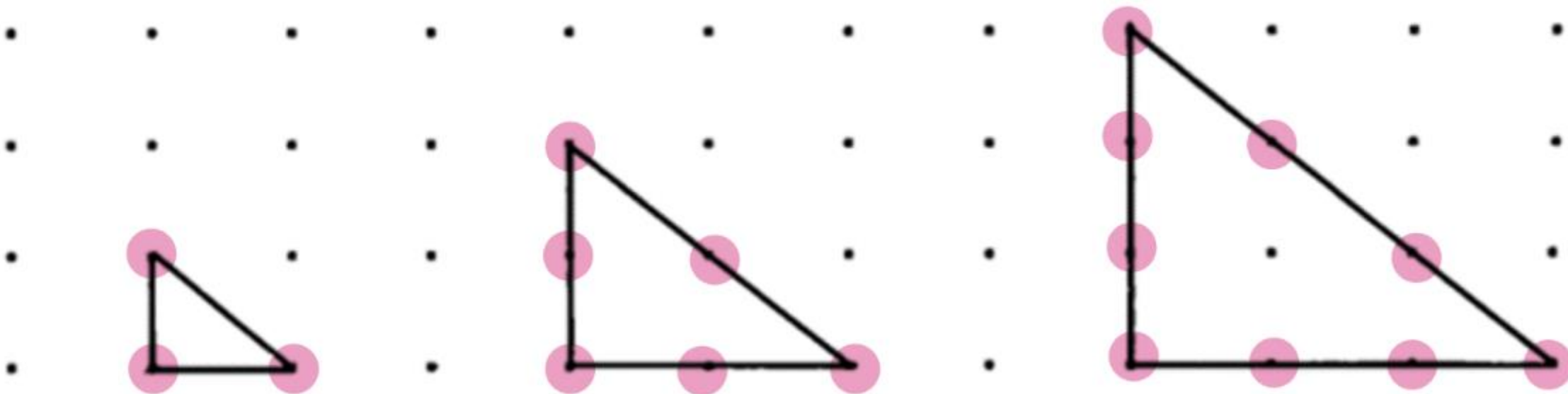


Learning Intentions

We are learning to:

- **investigate** numerical patterns

As this sequence of triangles and dots continues, investigate any patterns you can identify.

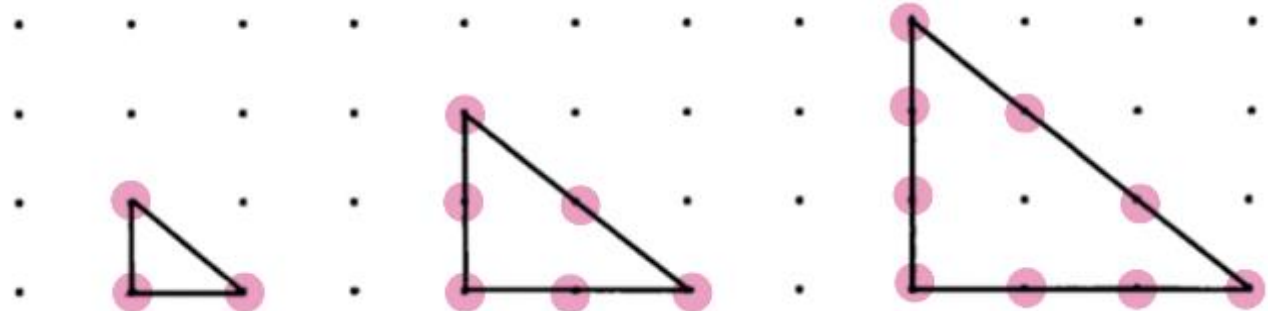




Success Criteria

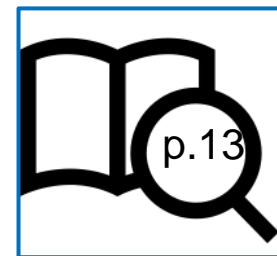
- I can count the number of dots on the perimeter of each triangle.
- I can count the number of dots inside each triangle.
- I can **sketch** the next triangle in this sequence.
- I can **classify** numerical patterns.
- I can **generalise** numerical patterns.
- I can **verify** my generalisation.

As this sequence of triangles and dots continues, investigate any patterns you can identify.



Extension:

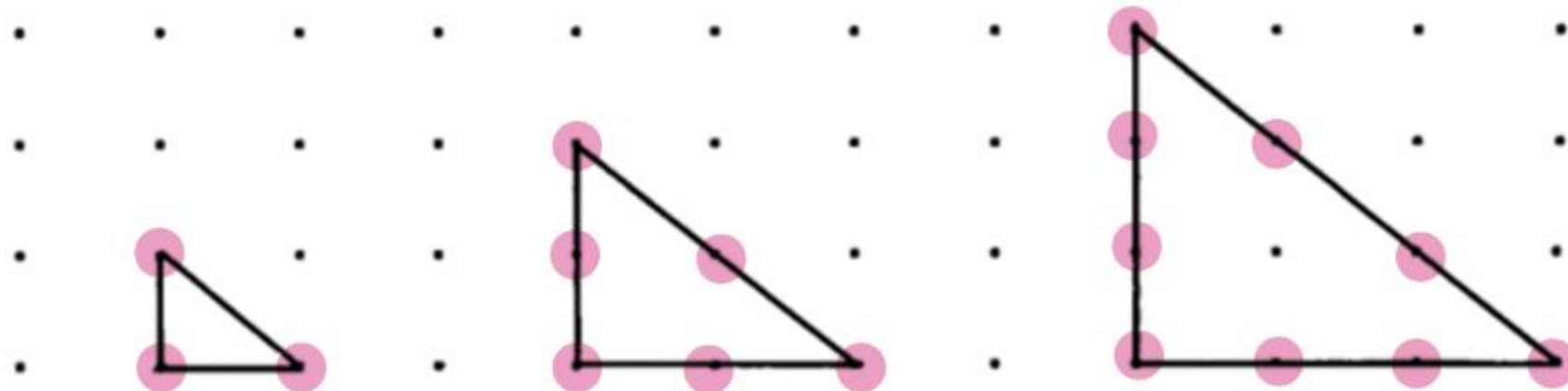
Michelle thinks the perimeter increases by 3cm for each successive triangle. Sarah disagrees. Who do you agree with? Explain your reasoning.





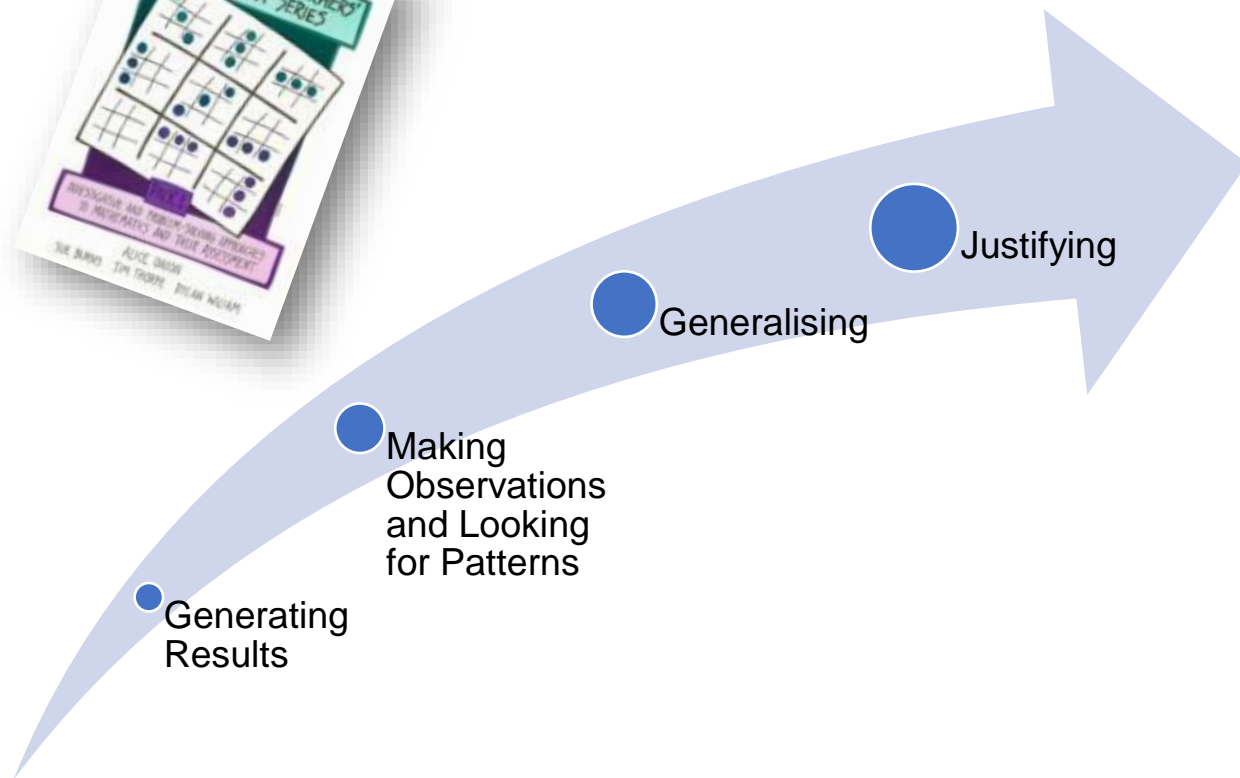
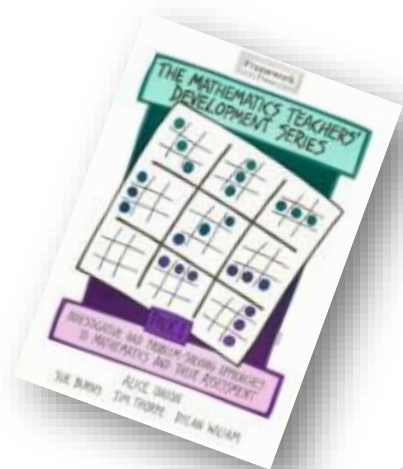
Investigative Activity

As this sequence of triangles and dots continues, investigate any patterns you can identify.





Investigative Work





Investigative Work

‘The process of completing the Classroom-Based Assessment should be viewed as part of teaching and learning, and not solely for assessment purposes’.

Guidelines for the Classroom-Based Assessments and Assessment Task p.8





Activity

Engage with an investigative task

15 mins



Hear from a teacher and observe students

10 mins



Discussion on investigative tasks

10 mins





Investigative Work



Can we identify elements of investigative work such as:

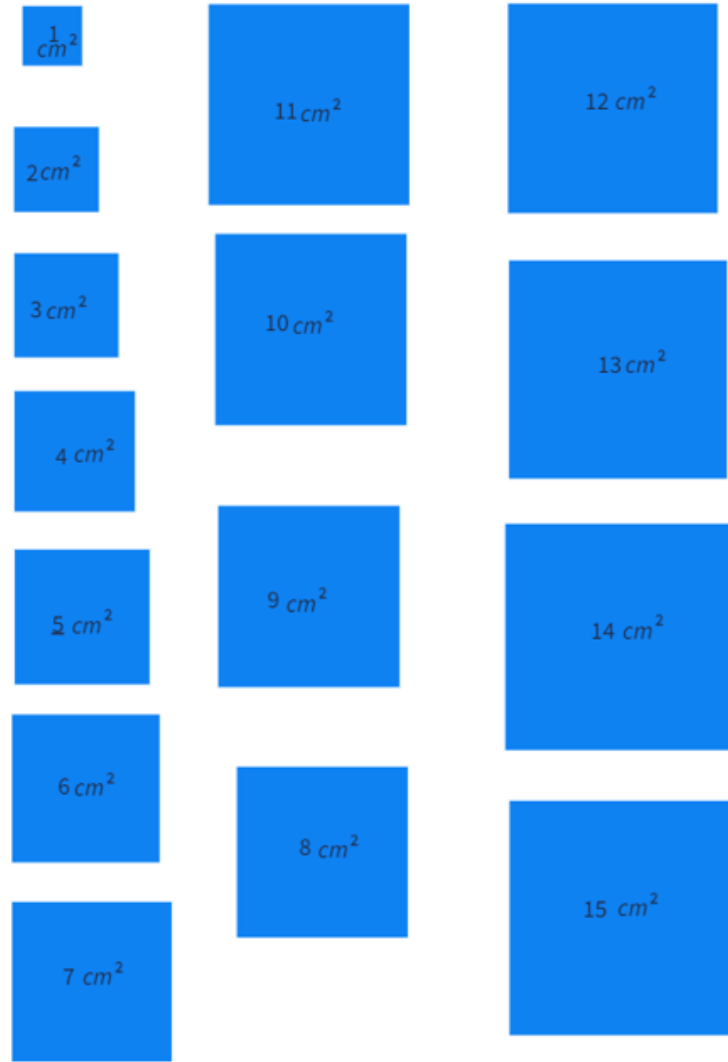
- generating results
- looking for patterns
- generalising
- justifying



Learning Intentions

We are learning to investigate relationships between the side lengths of triangles.

Investigate what triangles can you form using side lengths from the squares (without overlapping squares)?





Success Criteria

- I can create a triangle using the squares (without squares overlapping).
- I can measure the size of an angle.
- I can identify acute, right and obtuse angles.
- I can calculate the side length of a square given its area.
- I can generalise a relationship between side lengths of triangles.
- I can justify my generalisation.

Investigate what triangles can you form using side lengths from the squares (without overlapping squares)?



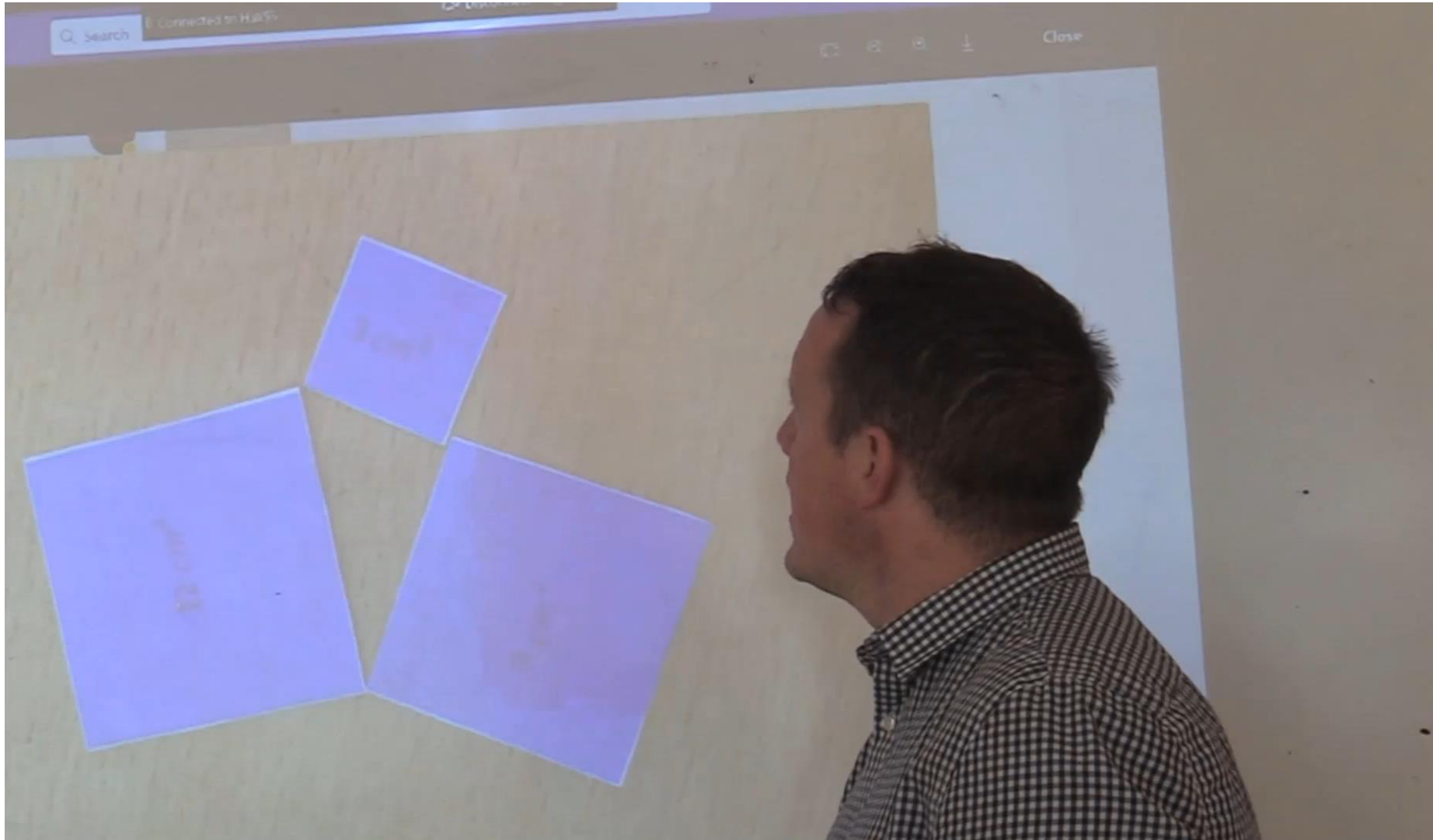
Observing Investigative Task



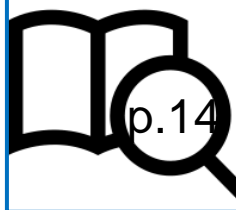
Success Criteria

- I can create a triangle using the squares (without squares overlapping).
- I can measure the size of an angle.
- I can identify acute, right and obtuse angles.
- I can calculate the side length of a square given its area.
- I can generalise a relationship between side lengths of triangles.
- I can justify my generalisation.

Teacher's Reflection on Lesson



Have we seen any evidence that would inform our planning for future lessons?





Activity

Engage with an
investigative task

10 mins



Hear from a teacher
and observe
students

10 mins



Discussion on
investigative tasks

10 mins





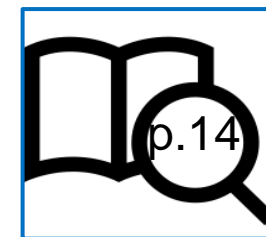
Share our thinking

Discussion on investigative tasks



Discuss:

1. Can you select or adapt an investigative task to use in class with your students in the next 2-3 weeks?
2. What Action Verbs might you use in the Success Criteria for this task?



10 mins

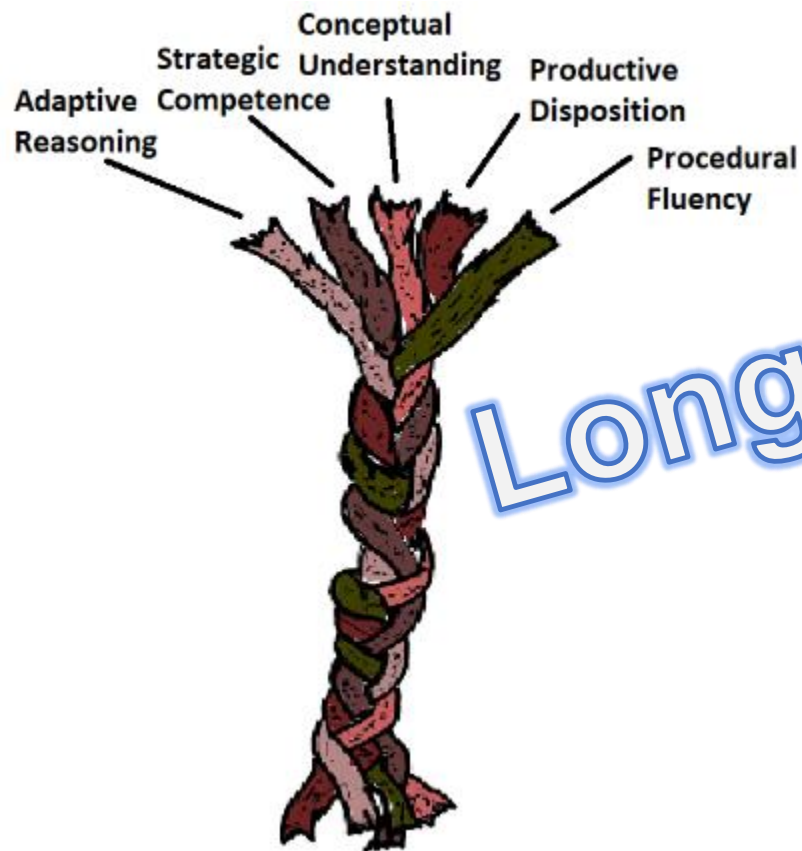


Learning Intention:

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Success Criteria:

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Aim for our students

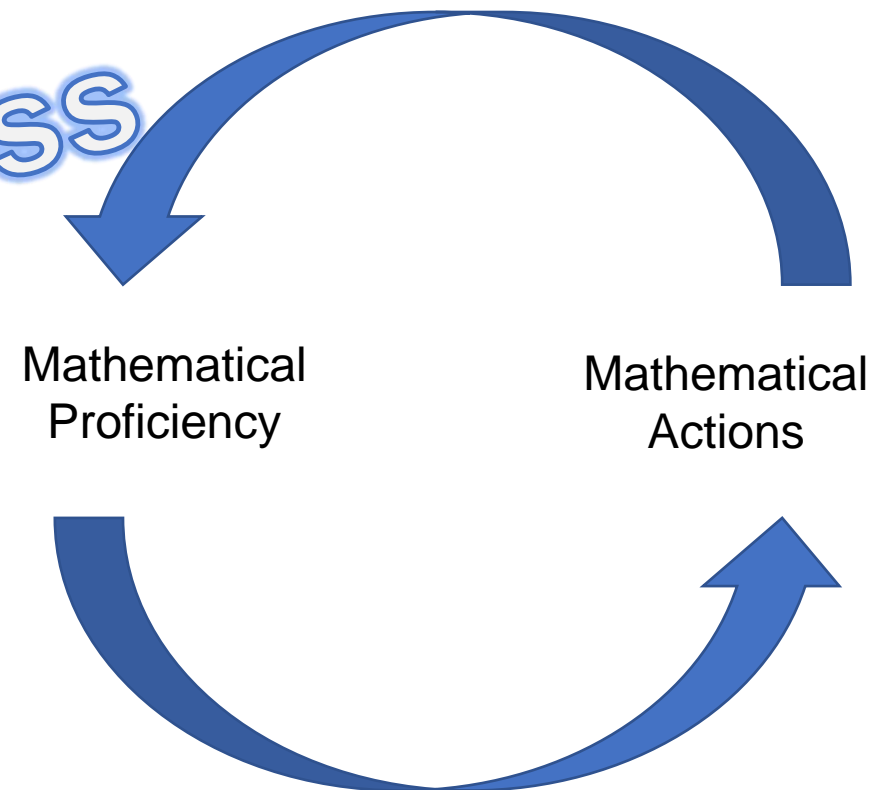
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Define	state a rule that identifies the elements of a set
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Justify	give valid reasons or evidence to support an answer or conclusion
Mathematise	generate a mathematical representation (e.g. graph, equation, geometric figure) to describe a particular aspect of a phenomenon

Process

Long term process



Interdependent



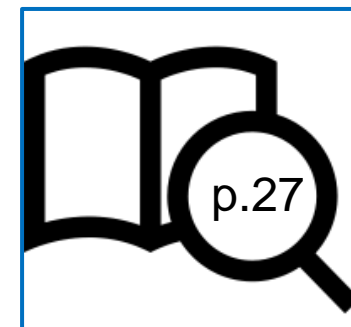
Reflective Log

What have I experienced that may enhance my students' experience?

How does it work in this context?

How might it work in my context?

How will I adapt/modify it for my practice?



5 mins



Oide

Tacú leis an bhFoghlaim
Ghairmiúil i measc Ceannairí
Scoile agus Múinteoirí

Lunch



1 hour





Outline for the day

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Oide

Tacú leis an bhFoghlaim
Ghairmiúil i measc Ceannairí
Scoile agus Múinteoirí

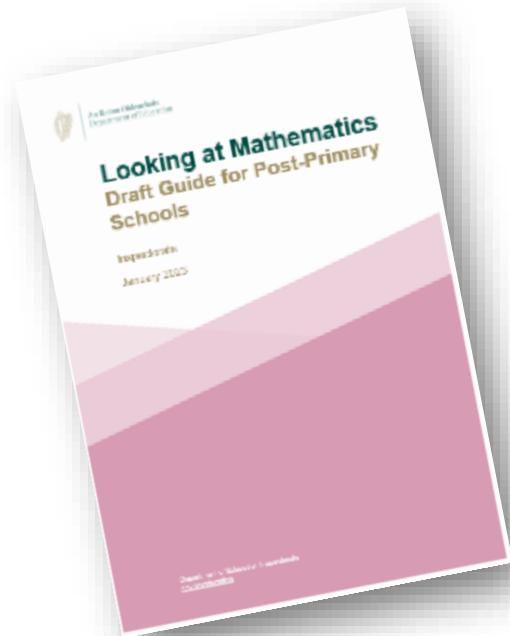
Supporting the Professional
Learning of School Leaders
and Teachers

Incorporating Rich Tasks into Classroom Practice



Learning Intention

To further develop the incorporation of rich tasks into classroom practice.



Students engage in **purposeful tasks** that link to their **real-world experiences**; students participate in **classroom discussions** and place value on listening and contributing.

(Looking at Mathematics, p.5)



Learning Intention:

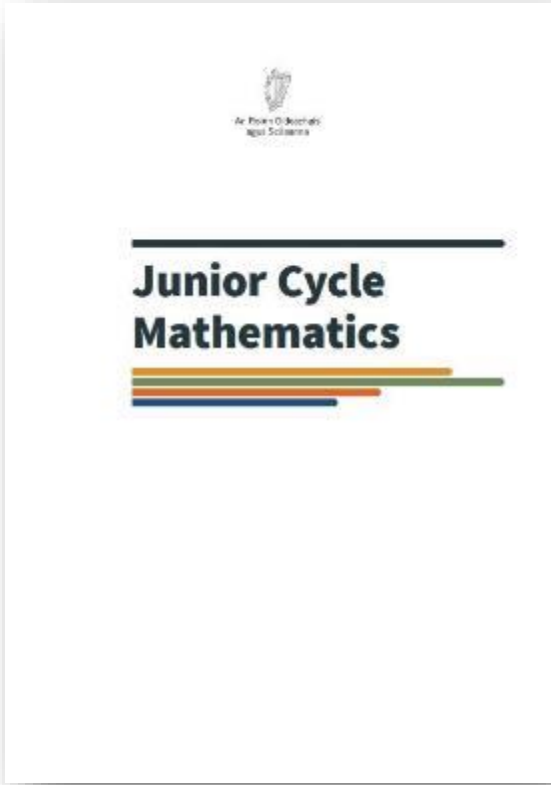
To further develop the incorporation of rich tasks into classroom practice.

Success Criteria:

- I can identify what is important in a task.
- I have reflected on my use of tasks
- I have created/selected resources to further develop my use of tasks
- I can sequence tasks with the student at the centre of the learning to develop understanding.



Rationale



**Junior Cycle
Mathematics**

Students engage in problem-solving tasks to use their mathematical skills in novel ways.

They reflect on their learning and compare their approaches to the strengths and weaknesses of their own and others' approaches.

From an early stage in the course, students should be encouraged to engage in a problem-solving cycle. This cycle involves students using successful problem-solving strategies in assessments throughout the year.

The quality of the tasks that learners engage with plays an important role in a problem-solving environment..... Problem-solving tasks activate creative mathematical thinking processes as opposed to imitative thinking processes activated by routine tasks. (Leaving Certificate Syllabus)

NCCA

September 2018

For examination from 2015

Department of Education
Government of Ireland

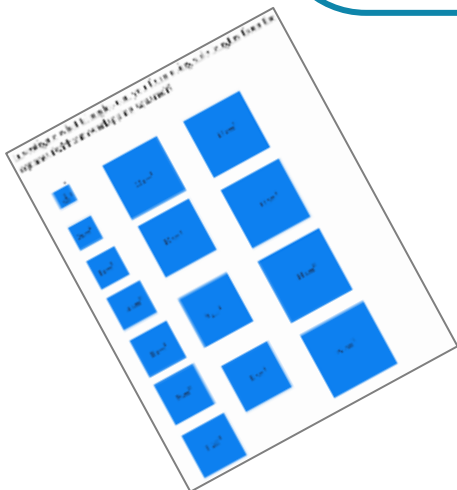
NCCA
National Council for Curriculum and Assessment



What is a rich task?

“....much of what it takes to make a rich task “rich” is the environment in which it is presented, which includes the support and questions that are used by the teacher and the roles that learners are encouraged to adopt.”

Jennifer Piggott, Rich Tasks and Contexts (<http://nrich.maths.org/5662>)



Right Angled Possibilities

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the length of the third side?

Can you explain your reasoning?

‘Natural numbers can be expressed as the sum of consecutive positive whole numbers.’

Is this statement *always, sometimes or never* true?



What are the characteristics of a rich task?





Characteristics of a rich task

Engaging

Provides access and challenge

Reasoning and generalisation

Discussion and communication

Decision making

Connecting strands

Creative

Multiple entry points



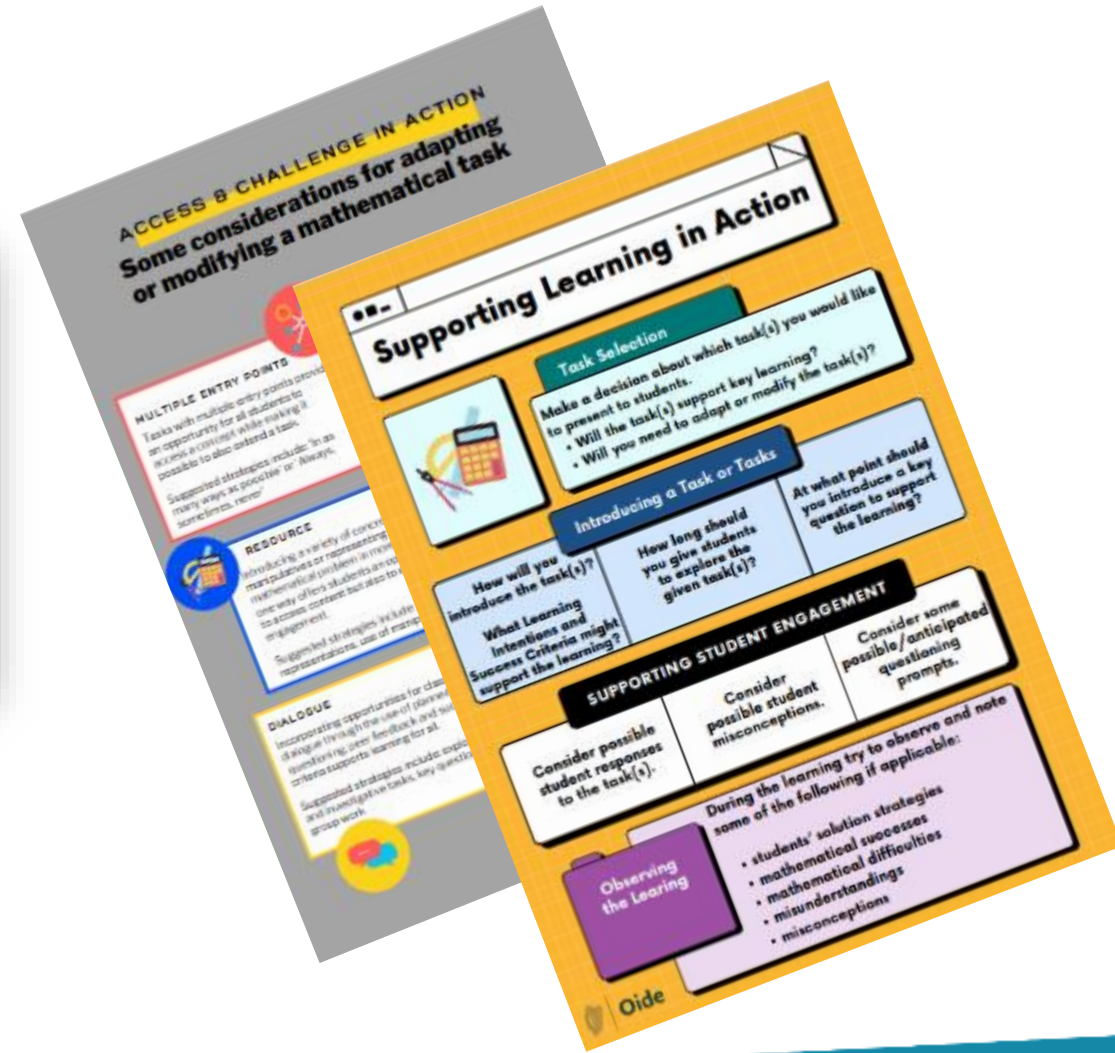
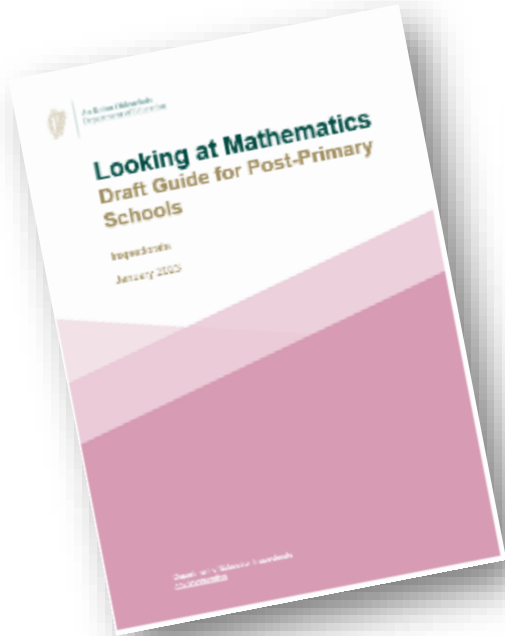
Teacher Voice



“they can see that the tools ..they are building up in their toolbox can then be used ...in different scenarios and in different problems



Factors to consider when choosing a rich task



Right Angled Possibilities

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the length of the third side?

Can you explain your reasoning?



Activity

Engage in self-reflection



10 mins

Looking at sequencing



35 mins

Sharing our thinking



10 mins



Self-Reflection

Engage in self-reflection



Getting Started

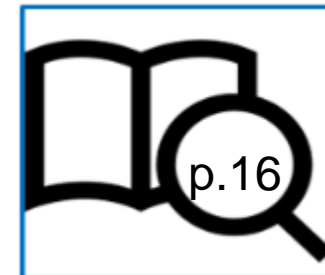
- I select, adapt and use tasks which support the key learning for all students
- I use tasks throughout a unit of learning

Up and Running

- I select a balance of tasks to build a range of mathematical skills and develop each of the components of mathematical proficiency
- I use learning intentions and success criteria to support student learning

Moving Forward

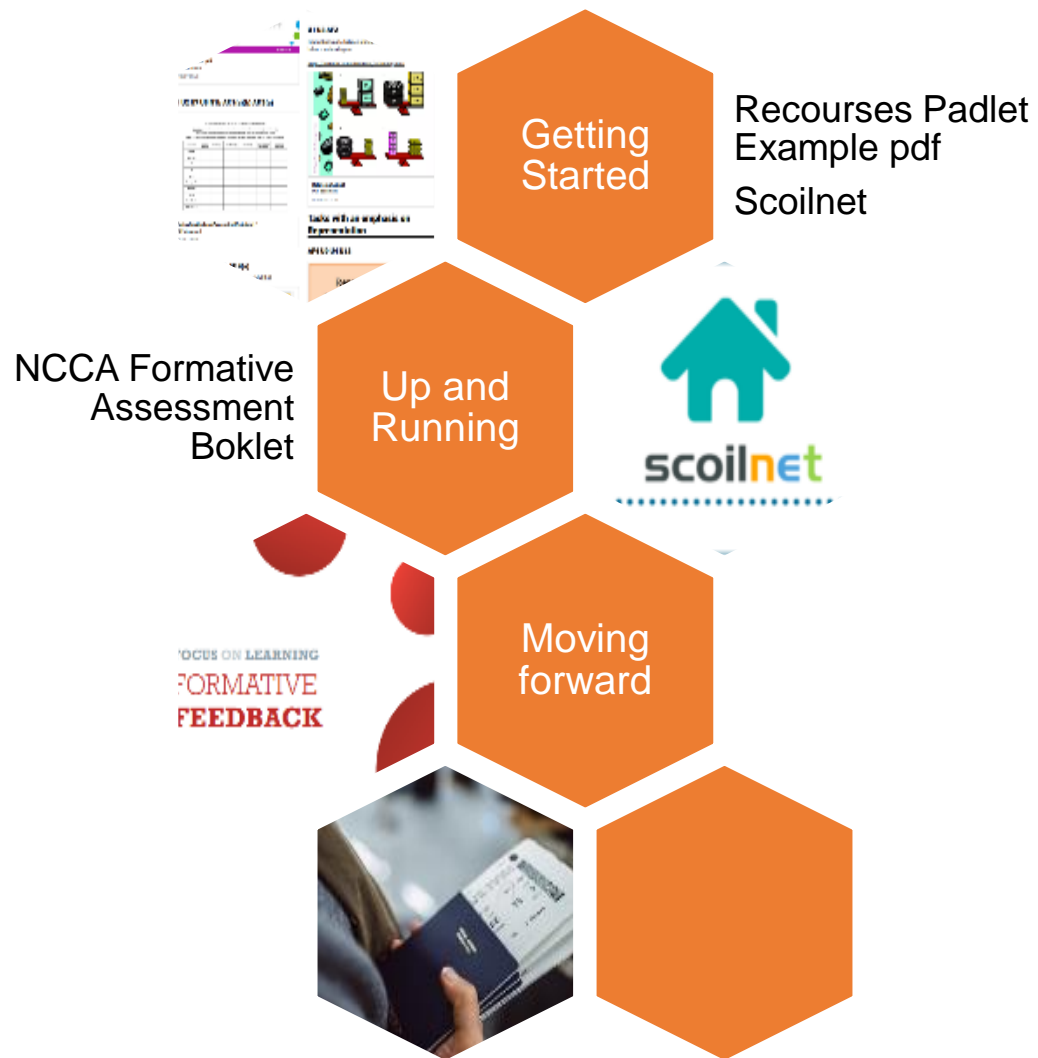
- I sequence tasks to teach for understanding
- I use tasks that develop student's prior knowledge to move them to new understandings



10 mins

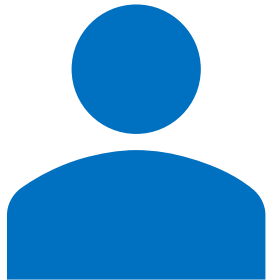


Resources





Engage in
self-reflection



10 mins

Looking at
sequencing



35 mins

Sharing our thinking



10 mins



Considerations for Sequencing

Assessment Practices:

- Is task-based learning included in your department assessment policies?
- Can you align your assessment of tasks to learning intentions and success criteria?
- Can you apply some of the language and the features of quality from the Junior Cycle Assessment guidelines to your assessment of rich tasks?

Subject
Department



Setting the Context

Learning Outcomes

GT3, GT4, U1, U2, U4, U5, U6, U7, U8, U9, U10, U11, U13

Context

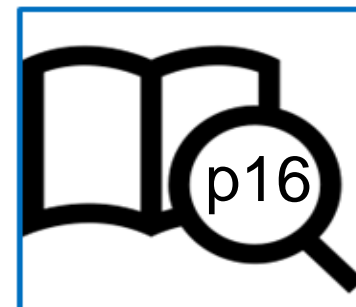
Consider your own context

Key Learning

- Students develop their mathematical arguments about geometric relationships
- Students apply Pythagoras' Theorem to solve right angle triangle problems
- Students analyse and interpret problems involving Pythagoras' Theorem
- Students develop their understanding of the construction of right-angled triangles

Ongoing Assessment

- Can students solve problems using Pythagoras Theorem in familiar and unfamiliar contexts?
- Can students develop a mathematical argument in relation to Pythagoras' Theorem?
- Can students select, justify and apply suitable constructions to solve problems involving right angled triangles?



Choose and sequence appropriate tasks



Incorporating Tasks



“we are looking at whether we are using a task at the beginning of the unit of learning to introduce a topic,using multiple tasks within the unit it’s something we are developing



Possible talking points

How can we start this unit with a task that fosters student engagement?

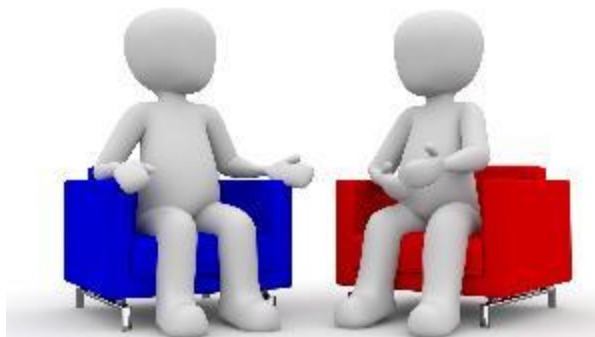
Can we select a range of tasks that incorporate a balance of the components of Mathematical Proficiency?

Do we have tasks that develop student's prior knowledge to move them to new understandings?





Looking at sequencing



Examine the key learning and ongoing assessment from the unit of learning

Take time to have a look at the rich tasks associated with the unit of learning

Take the tasks provided and sequence them, keeping in mind your own school context



Planning Support for a Unit of Learning

Learning Outcomes

GT3, GT4, U1, U2, U4, U5, U6, U7, U8, U9, U10, U11, U13

Context

Consider your own context

Key Learning

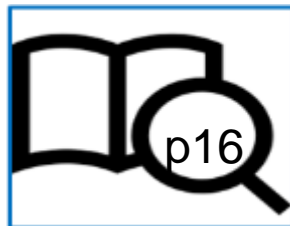
- Students develop their mathematical arguments about geometric relationships
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- Students analyse and interpret problems involving Pythagoras' Theorem
- Students develop their understanding of the construction of right-angled triangles

Ongoing Assessment

- Can students solve problems using Pythagoras Theorem in familiar and unfamiliar contexts?
- Can students develop a mathematical argument in relation to Pythagoras' Theorem?
- Can students select, justify and apply suitable constructions to solve problems involving right angled triangles?



5 mins





Supporting Tasks

Task C

Investigating Triangles

Investigate what triangles can you form using side lengths from the squares (without overlapping squares)?

1 cm^2	11 cm^2	12 cm^2
2 cm^2		
3 cm^2	10 cm^2	13 cm^2
4 cm^2		
5 cm^2	9 cm^2	14 cm^2
6 cm^2	8 cm^2	
7 cm^2		15 cm^2

Task B

Right Angled Possibilities

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the length of the third side?

Can you explain your reasoning?

Task K

Order the statements below to correctly prove Pythagoras' Theorem for given triangle ABC.



Figure 19.

Theorem 14 (Pythagoras). In a right-angle triangle the square of the hypotenuse is the sum of the squares of the other two sides.

Proof. Let $\triangle ABC$ have a right angle at B . Draw the perpendicular BD from the vertex B to the hypotenuse AC (shown in Figure 19).

The right-angle triangles $\triangle ABC$ and $\triangle ADB$ have a common angle at A . $\therefore \triangle ABC$ is similar to $\triangle ADB$.

$$\frac{|AC|}{|AB|} = \frac{|AB|}{|AD|}$$

so $|AB|^2 = |AC| \cdot |AD|$

Similarly, $\triangle ABC$ is similar to $\triangle BDC$.

$$\therefore \frac{|AC|}{|BC|} = \frac{|BC|}{|DC|}$$

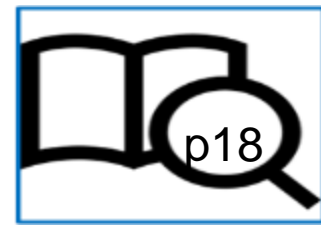
so $|BC|^2 = |AC| \cdot |DC|$

Thus $|AB|^2 + |BC|^2 = |AC| \cdot |AD| + |AC| \cdot |DC| = |AC| (|AD| + |DC|) = |AC| \cdot |AC| = |AC|^2$

Task A

Pythagorean Shell

What missing side lengths can you find in this diagram?



10 mins



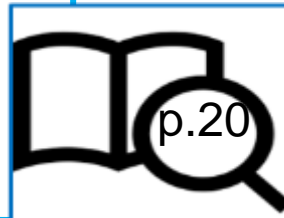
Activity

- Examine the tasks while reviewing the key learning
- Consider your own student context
- **Sequence the tasks in a way that makes most sense for your students**



While sequencing the tasks discuss:

- Which tasks are you choosing?
- How have you decided to sequence the tasks?
- What other learning is needed between tasks?
- How do mathematical ideas from the unit develop in this lesson sequence?





Supporting Tasks

Task C

Investigating Triangles

Investigate what triangles can you form using side lengths from the squares (without overlapping squares)?



Task B

Right Angled Possibilities

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the length of the third side?

Can you explain your reasoning?

Task A

Pythagorean Shell

What missing side lengths can you find in this diagram?



Task K

Order the statements below to correctly prove Pythagoras' Theorem for given triangle ABC.



Figure 19.

Theorem 14 (Pythagoras). In a right-angle triangle the square of the hypotenuse is the sum of the squares of the other two sides.

Proof. Let $\triangle ABC$ have a right angle at B . Draw the perpendicular BD from the vertex B to the hypotenuse AC (shown in Figure 19).

The right-angle triangles $\triangle ABC$ and $\triangle ADB$ have a common angle at A . $\therefore \triangle ABC$ is similar to $\triangle ADB$.

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$$\therefore \frac{|AC|}{|BC|} = \frac{|BC|}{|DC|}$$

so $|BC|^2 = |AC| \cdot |DC|$

Thus $|AB|^2 + |BC|^2 = |AC| \cdot |AD| + |AC| \cdot |DC|$
 $= |AC|(|AD| + |DC|)$
 $= |AC| \cdot |AC|$
 $= |AC|^2$



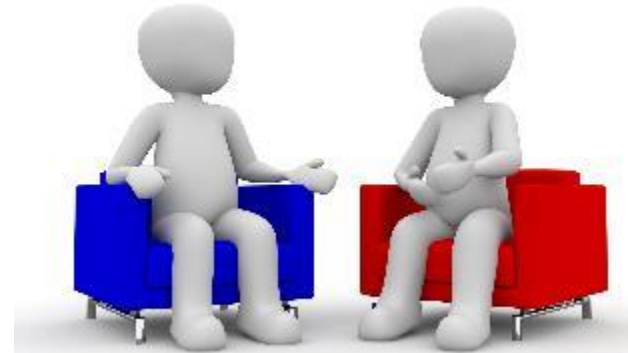
Sequencing

Which tasks are you choosing?

How have you decided to sequence the tasks?

What other learning is needed between tasks?

How do mathematical ideas from the unit develop in this lesson sequence?



Learning Outcomes
GT3, GT4, U1, U2, U4, U5, U6, U7, U8, U9, U10, U11, U13

Context
Consider your own context

Key Learning

- Students develop their mathematical arguments about geometric relationships
- Students apply Pythagoras' Theorem to solve right angle triangle problems
- Students analyse and interpret problems involving Pythagoras' Theorem
- Students develop their understanding of the construction of right-angled triangles

Ongoing Assessment

- Can students solve problems using Pythagoras Theorem in familiar and unfamiliar contexts?
- Can students develop a mathematical argument in relation to Pythagoras' Theorem?
- Can students select, justify and apply suitable constructions to solve problems involving right angled triangles?



20 mins



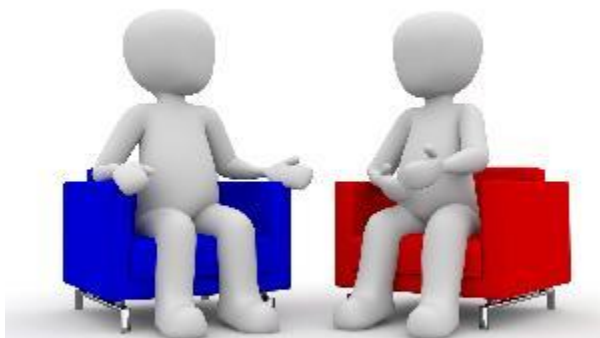
Activity

Engage in self-reflection



10 mins

Looking at sequencing



35 mins

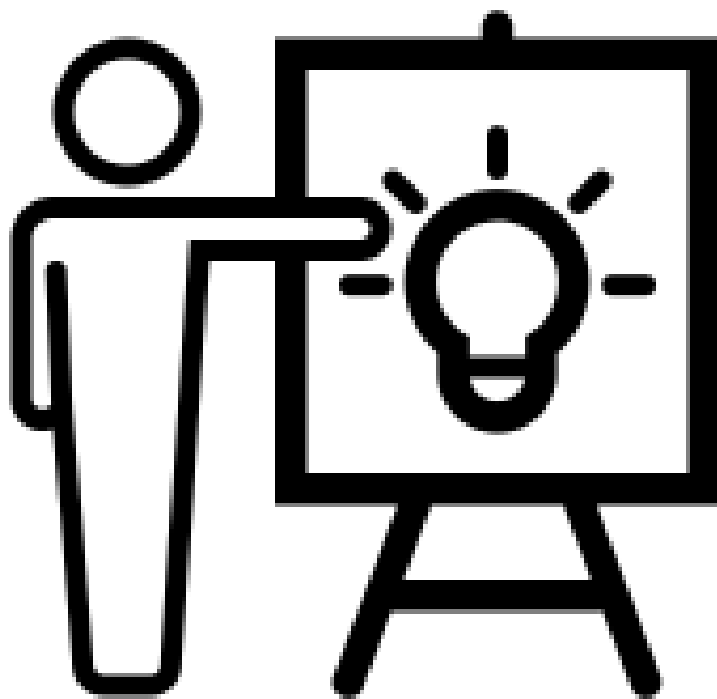
Sharing our thinking



10 mins



Sharing our thinking



Which tasks are you choosing?

How have you decided to sequence the tasks?

What other learning is needed between tasks?

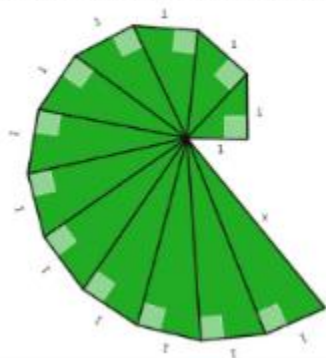
How do mathematical ideas from the unit develop in this lesson sequence?



Task A

Pythagorean Shell

What missing side lengths can you find in this diagram?



Task B

Right Angled Possibilities

If two of the sides of a right-angled triangle are 5cm and 6cm long, how many possibilities are there for the length of the third side?

Can you explain your reasoning?

Task C

Investigating Triangles

Investigate what triangles can you form using side lengths from the squares (without overlapping squares)?



Task D

Order the statements below to construct a Pythagorean Theorem for a right-angled triangle ABC.



Figure 19.

Theorem 14 (Pythagoras): In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.

Proof: Let $\triangle ABC$ have a right angle at B . Draw the perpendicular BD from the vertex B to the hypotenuse AC (shown in Figure 19).

The right-angled triangles $\triangle ABD$ and $\triangle CBD$ have a common angle at D . $\therefore \triangle ABC$ is similar to $\triangle ADB$.

$$\frac{|AC|}{|AB|} = \frac{|AD|}{|AB|}$$

$$\text{so } |AB|^2 = |AC| \cdot |AD|$$

Similarly, $\triangle ABC$ is similar to $\triangle BDC$.

$$\therefore \frac{|AC|}{|BC|} = \frac{|BC|}{|DC|}$$

$$\text{so } |BC|^2 = |AC| \cdot |DC|$$

$$\begin{aligned} \text{Then } |AB|^2 + |BC|^2 &= |AC| \cdot |AD| + |AC| \cdot |DC| \\ &= |AC| \cdot (|AD| + |DC|) \\ &= |AC| \cdot |AC| \\ &= |AC|^2. \end{aligned}$$

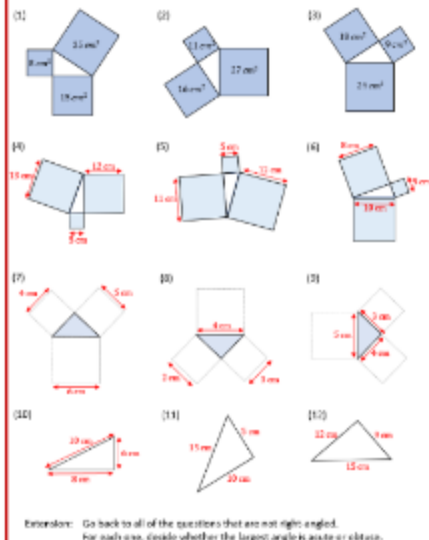
Task E

Which of the squares below will form a right-angled triangle if joined together at the vertices?



Task F

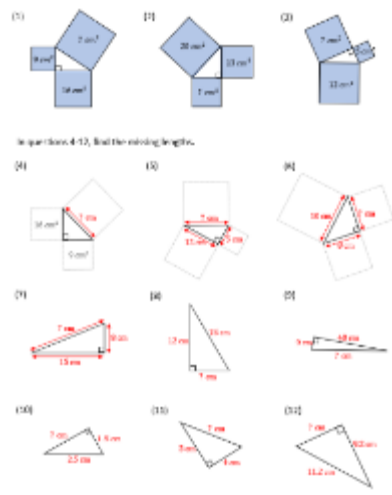
The diagrams below are not drawn accurately. For each question, decide whether the triangle is right-angled.



Extension: Go back to all of the questions that are not right-angled. For each one, decide whether the largest angle is acute or obtuse.

Task G

The diagrams below are not drawn accurately. Every triangle contains a right angle. In questions 1-3, find the missing areas. In questions 4-12, find the missing lengths.

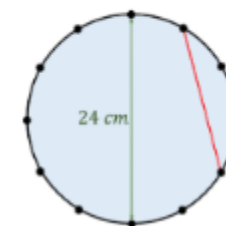


Task H

- Find the length of the diagonal.
- Calculate the area.
- A ladder leans against a wall. The ladder is 20m long. The bottom of the ladder is 3m from the wall. How far up the wall does the ladder reach?
- Find the distance two coordinates (2, 9) & (5, 12) are.
- The cylindrical pot below has a radius 5 cm. Would a 18 cm long pencil fit inside this pot? Calculate the height.

Task I

The spots are spaced equally around the circumference of the circle. Find the length of the red line.



Task J

Construct the following shapes accurately:

- A right-angled triangle with one angle 60° and the longest side being 9cm.
- A square with a diagonal of length 8cm.

Task K

Prove that the perpendiculars from a point on an angle bisector to the arms of the angle have equal length.



Learning Intention:

To further develop the incorporation of rich tasks into classroom practice.

Success Criteria:

- I can identify what is important in a task.
- I have reflected on my use of tasks
- I have created/selected resources to further develop my use of tasks
- I can sequence tasks with the student at the centre of the learning to develop understanding.



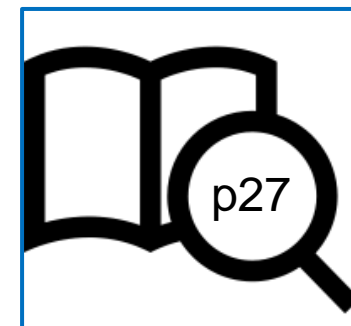
Reflective Log

What have I experienced that may enhance my students' experience?

How might it work in my context?

How does it work in this context?

How will I adapt/modify it for my practice?



3 mins



Oide

Tacú leis an bhFoghlaim
Ghairmiúil i measc Ceannairí
Scoile agus Múinteoirí

Supporting the Professional
Learning of School Leaders
and Teachers

Conclusion



Learning Intentions

- To identify and maximise opportunities for developing mathematical proficiency.
- To explore mathematical actions that support the development of mathematical proficiency.
- To discuss and reflect on how we support the incremental development of investigation skills for Junior Cycle Mathematics students.
- To further develop the incorporation of rich tasks into classroom practice.



Feedback

<https://registration.oide.ie/feedback>





Oide Mathematics Support

<https://dms.oide.ie/support/>

Principal can apply using

school roll number

password: schoolsupport2023





Oide

Tacú leis an bhFoghlaim
Ghairmiúil i measc Ceannairí
Scoile agus Múinteoirí

Supporting the Professional
Learning of School Leaders
and Teachers

Assessment Update



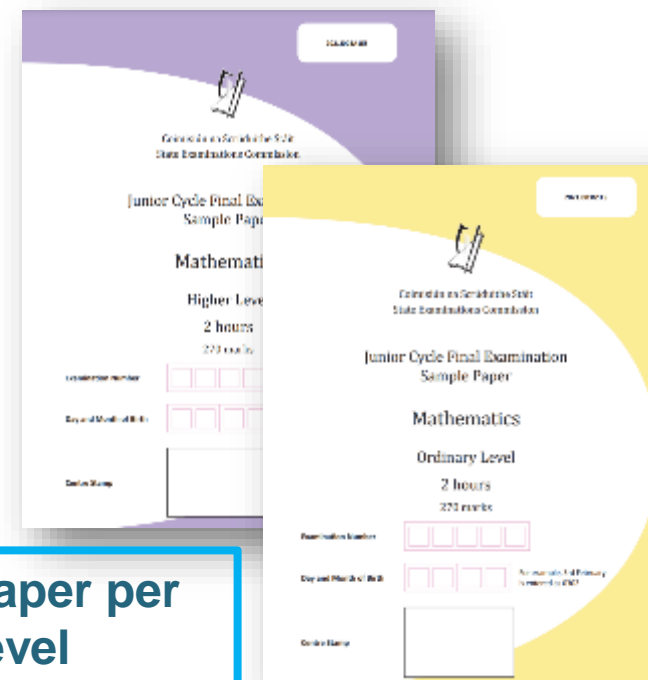
Assessment Information (2023/2024)

1.2 Key updates for the school year 2023/2024

Circular Number: 0028/2023

- It has been decided that the adjustments made to assessment arrangements for the 2022/2023 academic year will be maintained for one further year. As a result of these assessment adjustments, students entering third year in the school year 2023/2024 will be required to complete a minimum of one Classroom-Based Assessment rather than the usual two. Further detail is at section 3.5.
- This also means that students in third year in the school year 2023/24 will not be required to complete Assessment Tasks (ATs) in the relevant subjects.

The AT element will not be examined in 2024



One paper per level
Two hours duration



CBA - Third Year 2023/24



NCCA

An Chomhairle Náisiúnta
Curáil agus Measúnaithe
National Council for
Curriculum and Assessment

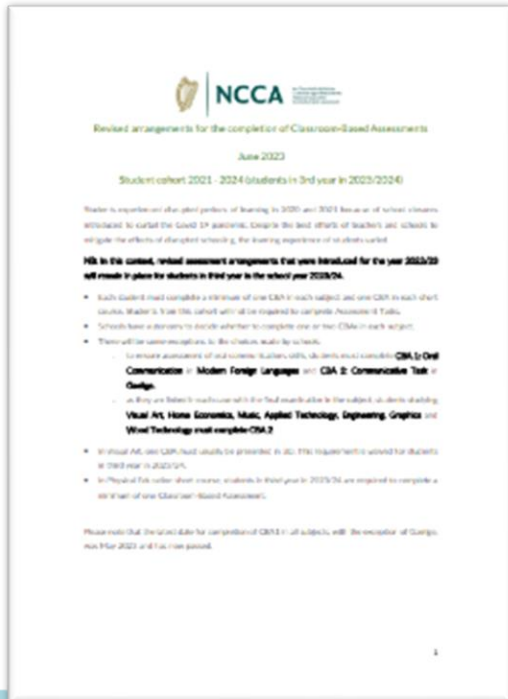
Revised arrangements for the completion of Classroom-Based Assessments

June 2023

Student cohort 2021 - 2024 (students in 3rd year in 2023/2024)

- Each student must complete a minimum of one CBA in each subject and one CBA in each short course. Students from this cohort will not be required to complete Assessment Tasks.
- Schools have autonomy to decide whether to complete one or two CBAs in each subject.

Please note that the latest date for completion of CBA1 in all subjects, with the exception of Gaeilge, was May 2023 and has now passed.



Key Dates		
Current Third Years	Classroom-Based Assessment 2 in English, Science, Business Studies, Modern-Foreign Languages, Mathematics, History, Geography, Classics, Religious Education, Jewish Studies	
	Latest date for completion	Thurs 14 th March 2024
	Latest date for award of provisional descriptors by the teacher	Mon 25 th March 2024
	Latest date for completion of Subject Learning and Assessment Review and for award of final descriptors by the teacher	Fri 29 th March 2024

The information on this slide is taken from the NCCA release of June, 2023, please check www.curriculumonline.ie for updated information



CBA - Second Year 23/24



NCCA An Chomhairle Náisiúnta
Curriculum, Assessment &
National Council for
Classroom and Assessment

Revised arrangements for the completion of Classroom-Based Assessments

June 2023

Student cohort 2021 - 2024 (students in 3rd year in 2023/2024)

Student Cohort 2022– 2025 (2nd years in 2023/2024)

Arrangements for this cohort remain as per the *Framework for Junior Cycle (2015)*.

Key Dates

Current Second Years

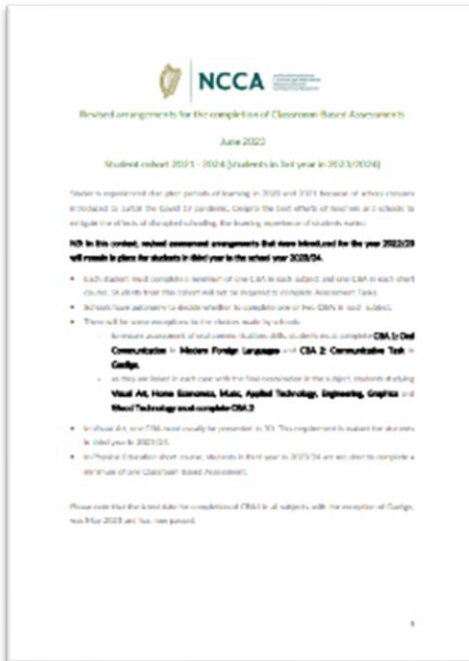
The information on this slide is taken from the NCCA release of June 2023, please check www.curriculumonline.ie for updated information

Student Cohort 2022– 2025 (2nd years in 2023/2024)

Arrangements for this cohort remain as per the *Framework for Junior Cycle (2015)*.

Classroom-Based Assessment 1 in all subjects

Window for completion	Monday 30 th October 2023 – Friday 10 ^h May 2024
Latest date for award of provisional descriptors by the teacher	Wednesday 15 th May 2024
Latest date for completion of Subject Learning and Assessment Review and for award of final descriptors by the teacher	Wednesday 22 nd May 2024







References

- DES. (2023) Looking at Mathematics: Draft Guide for Post-Primary Schools.
- DES. (2022). Looking at our Schools 2022: A Quality framework for Post-Primary Schools. Dublin
- Kilpatrick et al., (2001). Adding it Up: Helping Children Learn Mathematics.
- Mason, J. & Johnson-Wilder, S. (2004). Designing and Using Mathematical Tasks.
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- NCCA. (2019) Guidelines for Classroom Based Assessments and Assessment Task
- Onion et al., (1990) Investigative and Problem-Solving Approaches to Mathematics and their Assessment
- Piggott. J, Rich Tasks and Contexts (<http://nrich.maths.org/5662>)
- ponderingplanning.wordpress.com/author/ponderingplanning/