



**Oide**

Tacú leis an bhFoghlaim  
Ghairmiúil i measc Ceannairí  
Scoile agus Múinteoirí

Supporting the Professional  
Learning of School Leaders  
and Teachers

# Leaving Certificate Applied Mathematics

National Seminar 10  
Professional Learning Booklet  
2023-2024





# Schedule

|               |   |
|---------------|---|
| 09:30 - 11:00 | A Kinematic Study of a Stretched/Compressed Spring  |
| 11:00 - 11:15 | Tea and Coffee                                      |
| 11:15 - 13:00 | Video Analysis, a Valuable Teaching & Learning Tool |
| 13:00 - 14:00 | Lunch   |
| 14:00 - 15:30 | Managing our Subject Planning                       |

## Key Messages

Core to the specification is a non-linear approach which will promote the making of connections between various learning outcomes.

Strand 1 is the unifying strand and emphasises the importance of utilising mathematical modelling across all learning outcomes.

Applied Mathematics is rooted in authentic problems as a context for learning about the application of Mathematics.

# Professional Development Supports

## Overview of Support to Date

- 9 National Seminars
- 4 Collaboratives
- 2 Technology Workshops

Slides and additional Resources available

- 4 Webinars
- Video resources

Recordings available online

# Professional Development Supports

## Overview of Upcoming Support

Year 4 September 2023 - May 2024



# A Kinematic Study of a Stretched/Compressed Spring

## By The End of This Session You Will Have:

Explored a student-centred approach to developing an understanding of this topic.

Gained further experience of the ‘Concepts through modelling’ approach to teaching and learning.

Experienced a **constructivist teaching approach** to actively involve students in investigating kinematic equations for spring movement.

# Concepts through Modelling

A trampoline is constructed with several elastic materials which can stretch and return to their original shape. As the elastic material moves, its potential and kinetic energies are continually changing.

Select any material in this image, which exhibits such plastic properties and can withstand repeated stress?

In groups,

- discuss the various quantities that students may consider, on viewing this image?
- consider any Laws that might come into play.



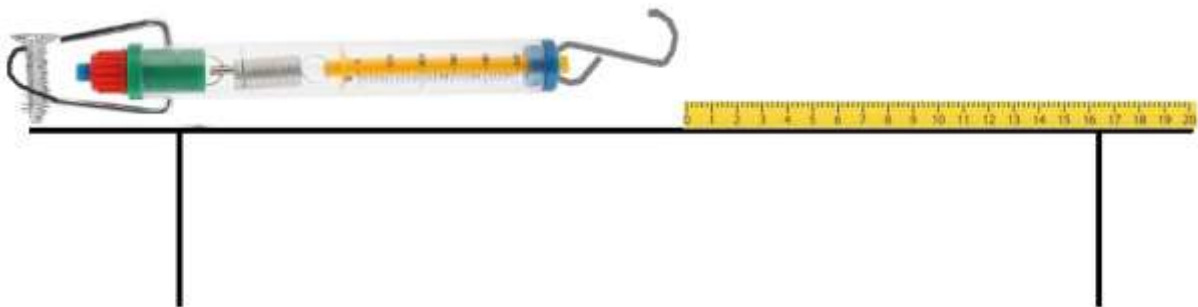
# Student Led Enquiry

How would you interpret the movement, of this two-dimensional representation, of the spring?



## Interpreting the spring movement?

Using the given apparatus, establish if a relationship exists between the force applied to a spring and the extension of the spring.



Can this concept be investigated or replicated at your tables?

Consider what causes the spring to extend and contract?

# Feedback

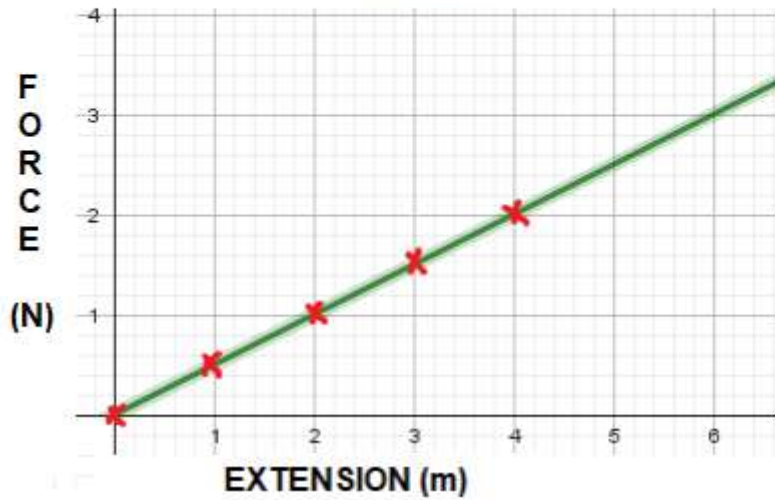
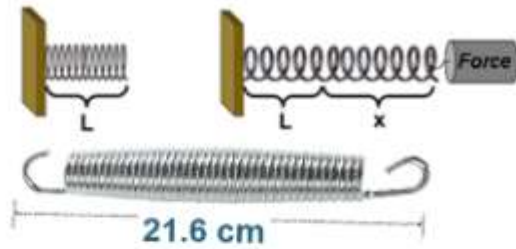
## Guided Discovery

A relationship exists between the force applied to the spring and the extension of the spring.

What assumptions did we make?



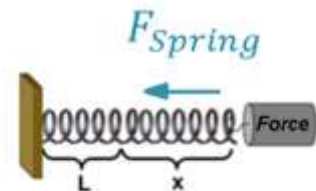




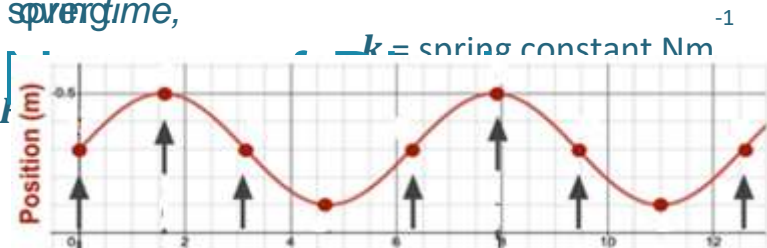
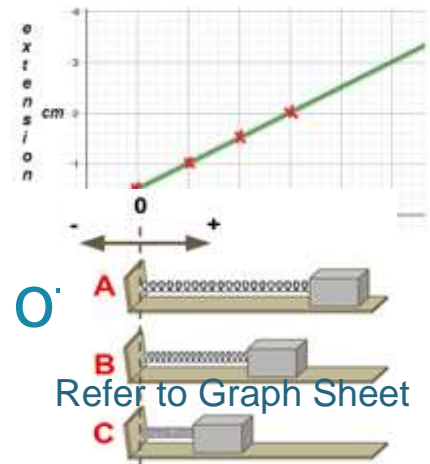
# Hooke's Law

## Horizontal Springs

The spring exerts a force in the opposite direction as the direction of its stretch (or compression).



Hooke's Law states that a linear relationship between the amount of stretch and the displacement of a spring. As the displacement of a spring changes periodically over time,



Refer to Graph Sheet

# Speed and Velocity

As the block vibrates back and forth, its speed changes.

The speed is 0 m/s at the extreme positions and a maximum value at the equilibrium position.

Moving from B to A *and* from B to C  
The block **slows down**.

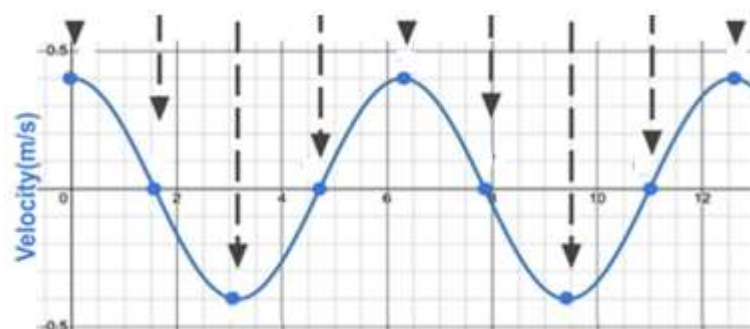
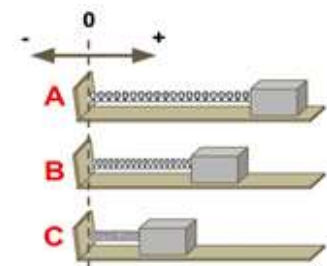
Moving from A to B *and* from C to B

The block **speeds up**.

## Nature of Velocity of a Spring

As the block vibrates back and forth, its velocity changes periodically over time.

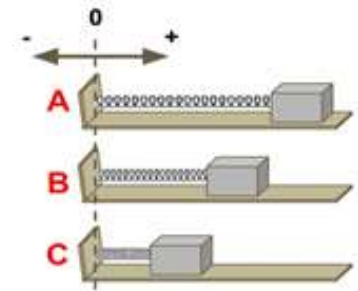
*IDENTIFY the positions of A, B & C on the velocity-time graph below.*



# Acceleration of a Spring

As the block vibrates back and forth, its acceleration changes periodically over time,

Using the graph template provided, IDENTIFY the acceleration at positions A, B & C.

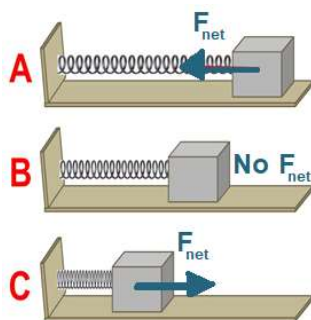


What can we conclude about the net force at each point?

# Nature of Acceleration of a Spring

As the block vibrates back and forth, its speed changes.

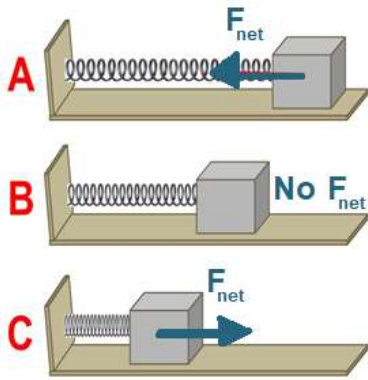
The speed is 0 m/s at the extreme positions and a maximum value at the equilibrium position.



The acceleration is in the direction of and proportional to the net force (restoring force).

$F_{net}$ , acceleration is always directed towards the equilibrium position.

$F_{net}$ , acceleration is largest at the extremes and 0 m/s/s at the equilibrium position.

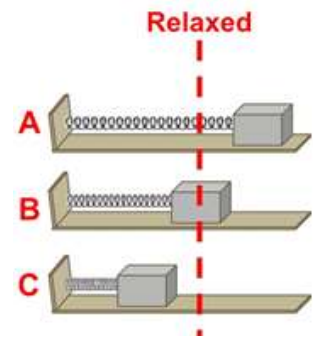


## Energy Analysis(Horizontal Springs)

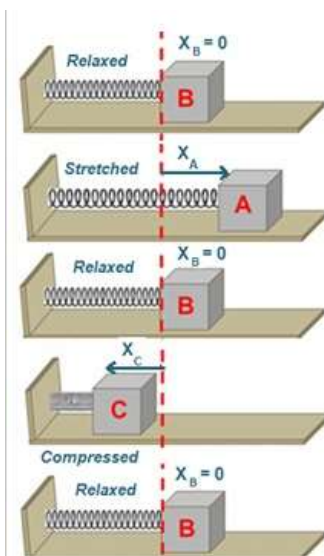
As the block vibrates back and forth between extremes, energy is changing from **Elastic Potential Energy** and **Kinetic Energy**.

**Kinetic energy** (speed dependent) is greatest at position B.

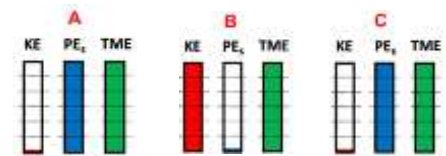
**Elastic Potential Energy** (stretch/compression dependent) is greatest at positions A and C



## Energy Analysis(Horizontal Springs)

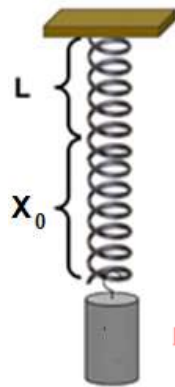
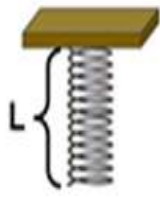


- B : PE<sub>spring</sub> and KE
- A : PE<sub>spring</sub> and KE
- B : PE<sub>spring</sub> and KE
- C : PE<sub>spring</sub> and KE
- B : PE<sub>spring</sub> and KE



As the **Kinetic Energy** increases, the **Elastic Potential Energy** decreases and vice versa, the **Total Mechanical Energy** remains constant.

# Hooke's Law (Vertical Springs)



$$\Sigma F = k \cdot X_0 - mg$$

But  $\Sigma F = 0$

$$\Rightarrow k \cdot X_0 - mg = 0$$

$$\Rightarrow k \cdot X_0 = mg$$

$$\Sigma F = k \cdot (X + X_0) - mg$$

$$\Rightarrow \Sigma F = k \cdot X + k \cdot X_0 - mg$$

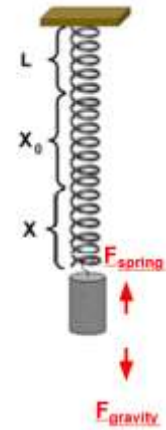
However,  $k \cdot X_0 = mg$

$$\Rightarrow \Sigma F = k \cdot X + k \cdot X_0 - mg$$

becomes

$$\Rightarrow \Sigma F = k \cdot X + mg - mg = k \cdot X$$

$$\Rightarrow \Sigma F = k \cdot X$$



So, the restoring force is  $k \cdot X$  as the spring stretches out  $X$ cm from its resting point.

## Concepts through Modelling Approach

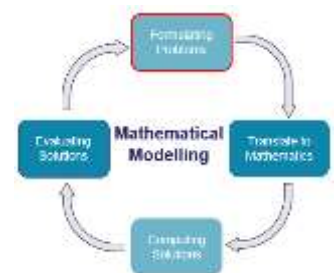
*A trampoline is constructed with a number of elastic materials which can stretch and return to their original shape. As the elastic material moves, its potential and kinetic energies are continually changing.*



*Select any material in this image, which exhibits such plastic properties and can withstand repeated stress?*

In groups,

- Consider how students might formulate variations to this problem?



# Reflection

How well did this session assist you in your understanding of how Hooke's Law can be developed and formalised through authentic modelling problems?

## Take Away Question

Consider the spring action on this Pinball machine.

What concepts from our specification might be developed concerning the movement of the ball, as a result of that spring motion?



# Using Video Analysis to Support Students' Engagement with Modelling

## By The End of This Session You Will Have:

Engaged with Video Analysis as a tool to gather, represent and interpret authentic real-world data.

Explored the use of Video Analysis as an enabler of understanding.

Investigated Video Analysis to enhance teaching and learning of Projectile Motion and other Strand 3 outcomes.



# What is Video Analysis?



Video Analysis obtains real-world position and time data from frames of a video which can then be analysed.

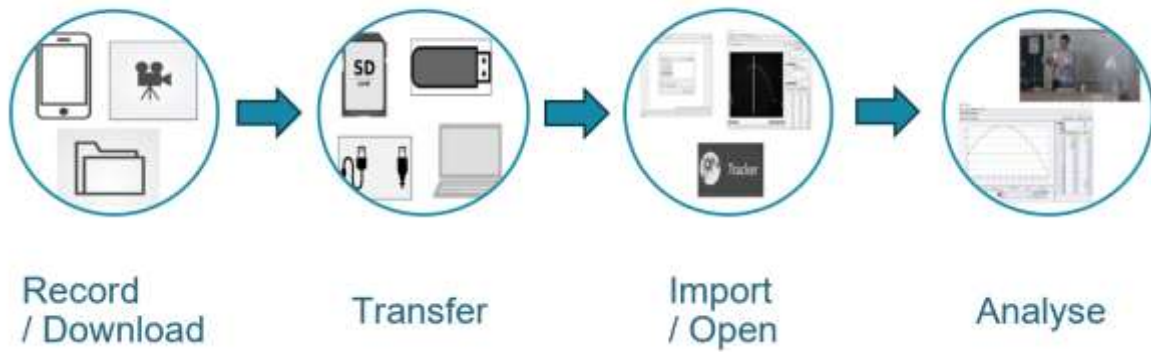
## Video Analysis Demonstration



Video Analysis carried out using “Tracker”, free software available for download from <https://physlets.org/tracker/>

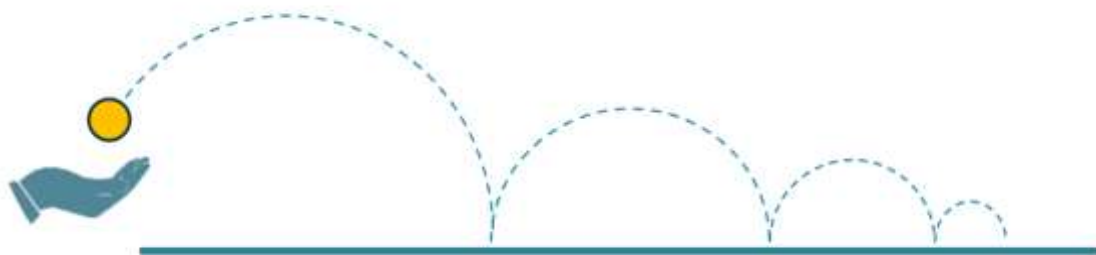


# Video Analysis Process



## Group Task

Record a short video clip of a ball thrown onto flat surface



# Record A Suitable Video



Use a high contrast background and/or brightly coloured object

Try not use very fast-moving objects if using standard cameras/smartphones

Keep the camera still while recording

Include a measurable item (e.g. metre stick) in the video

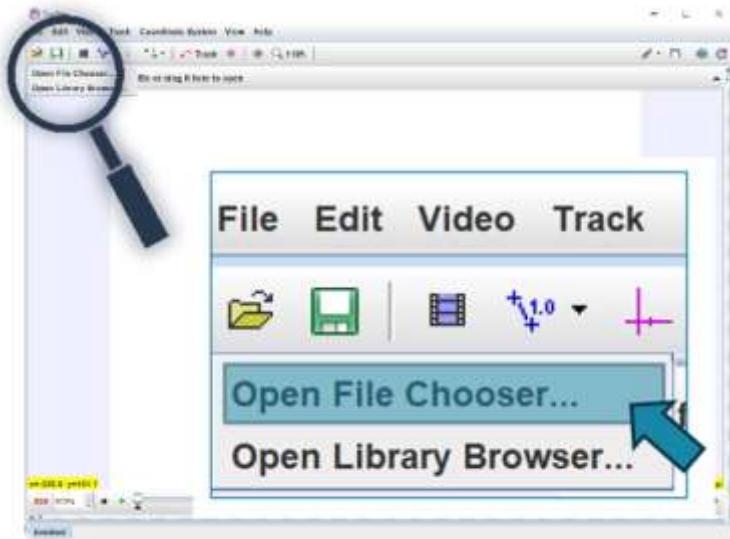
Set the camera up level and use motion that is perpendicular to the camera and from left to right



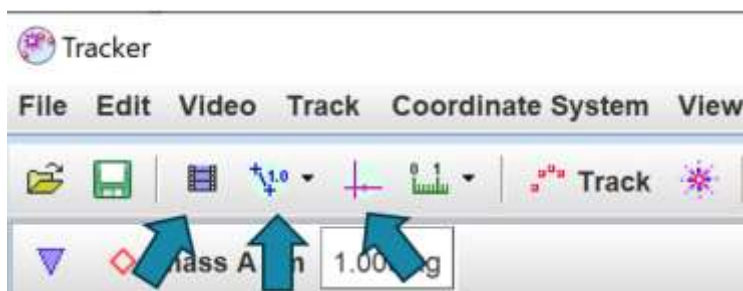
# Transfer Video



# Import Video File



# Analyse Video File



# Group Discussion

How could Video Analysis be useful in other contexts?

With what other topics/learning outcomes could you see it being helpful?



What challenges exist in using vide analysis in your classroom and how could these be overcome?

## Video Analysis and Applied Mathematics

Students should be able to use computational technology to solve problems Specification p. 16

It is anticipated that digital technology may be used as a learning tool in some aspects of this course Specification p. 6

It is expected that, in this course, students use digital technology for numerical experimentation and simulation Specification p.10

Modelling requires students to turn authentic situations into mathematical structures Specification p. 6

Accurately use mathematics to represent real-world phenomena, analyse and evaluate information and data from different sources Specification p. 10

# Aspects to consider when planning

|                                 |  |  |
|---------------------------------|--|--|
| Students' Prior Knowledge       | Links within Applied Maths Specification | Cross-Curricular Links                     |
| Real Life Examples/Applications | <b>Mathematical Modelling Cycle</b>      | Teaching, Learning & Assessment Approaches |
| Resources & Materials           | Learning Intentions & Success Criteria   | Inclusion                                  |

What do you want your students to get out of this?

## Real Life Applications Projectile Motion



"Applied Mathematics is inherently a transdisciplinary subject, authentic and relevant to the Real World"  
Specification p.8



# Resources and Materials

## Projectile Motion



"The course is experiential in its structure and emphasises the practical application of mathematical knowledge to the world around us." Specification p.13



## Reflection

What were your key takeaways from this session?

How can you implement ideas from this session in your teaching?

What are the next steps for enhancing teaching and learning using this technology?



# Planning for Teaching, Learning and Assessment

## By The End of This Session You Will Have:

Discussed/reflected on key learning from student engagement with the mathematical modelling cycle to inform your content and pedagogical planning.

Determined the need for allowing sufficient scope for change while developing a subject plan.

Worked in groups to plan a unit of work using a concepts through modelling approach.

# Rationale for Planning a timeline

When the brief for the project will be issued?

Will I now have to drop what I'm doing at the minute to cover circular motion?

I haven't done circular motion yet.

What topics should I have covered before Project is released?

What topics can I delay teaching ....

Is anybody else in the same boat as me?  
I usually teach Integration in January.

HELP!

I usually teach circular motion last and now I feel will I might struggle to teach it earlier in order to prepare my students for their project.

## Planning and Implementation Engagement with the Specification

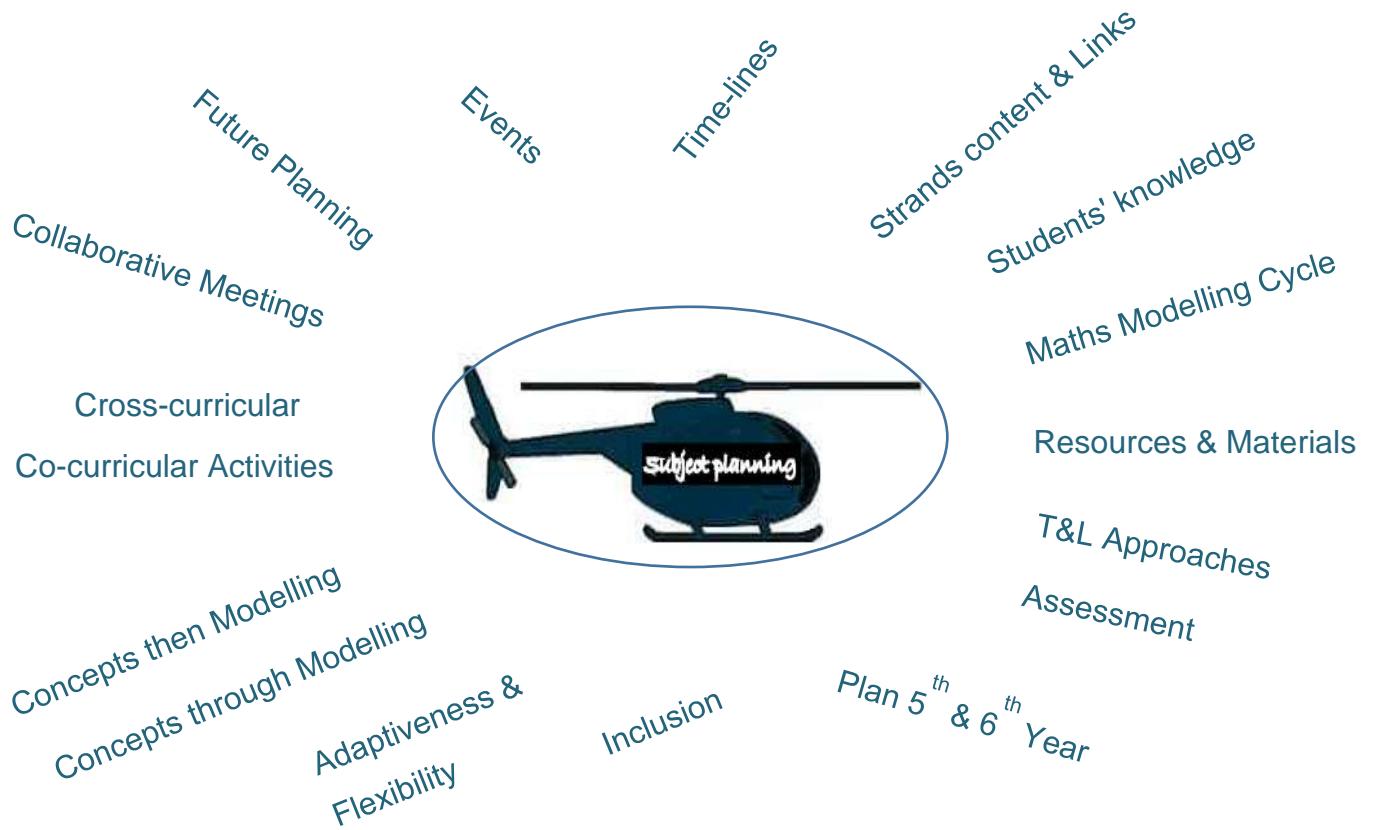
Now that the first two-year cycle is complete

- What does the term flexible planning mean to you?
- How might it be relevant for engaging with the Specification?





# Overall 'helicopter view' of our subject planning?



# Key questions when designing any *Unit of Work*?

What Learning outcomes will we include?

What prior knowledge learning should students have?

What other strands will this link to?

How many lessons would we anticipate this will this take?

When would be an appropriate time to engage with these learning outcomes?

What degree of flexibility could I build into planning this Unit of work?

## Individual – Planning a Unit of Work

Consider aspects from the planning grid, to assist you plan a unit of work using a *concepts through modelling* approach, for **Projectile Motion**.



# Feedback from Groups



# Sample Planning Unit of Work

| Topics                          | Learning Outcomes  | Prior Knowledge & Classwork   | Links & Cross-Curricular   | Success Criteria & Assessment   | Real-life applications & Resources  | Teaching & Learning Approaches   | Start Date<br>End Date |
|---------------------------------|--|---|--|---|---|--|------------------------|
| Projectiles<br>Horizontal Plane | Solve constant acceleration projectile motion problems involving displacement, velocity and time | <ul style="list-style-type: none"> <li>• Target Practice</li> <li>• Use of Symmetry</li> <li>• Time of Flight</li> <li>• Max. Height</li> <li>• Landing angles</li> <li>• Condition for maximum Range (Calculus 6<sup>th</sup> Yr)</li> </ul> | <ul style="list-style-type: none"> <li>• Solving Trigonometric identities</li> <li>• Solve problems using equations.</li> <li>• Identify conditions to be fulfilled for particular circumstances</li> <li>• Introducing wind resistance</li> </ul> | <ul style="list-style-type: none"> <li>• Recognising link to real world</li> <li>• Class Discussion</li> <li>• Class and homework exercises</li> <li>• Understanding difference between developing primary data and authenticating secondary data</li> <li>• End-of-topic Test</li> </ul> | <ul style="list-style-type: none"> <li>• Use of Rocket launcher and paper "rocket" projectiles to illustrate principles and stimulate interest in topic.</li> <li>• Use of <a href="#">phet</a> simulations.</li> <li>• <a href="#">Youtube</a> projects</li> <li>• Tracker software to collect real-time primary data</li> </ul> | <ul style="list-style-type: none"> <li>• Active Learning</li> <li>• Differentiated Instruction</li> <li>• Collaborative Learning</li> <li>• Experiential Learning</li> <li>• Project-Based Learning</li> </ul> |                        |

| L. C. Applied Maths: Student Learning Outcomes   |   |   |   |
|--|---|---|---|
| <b>Strand 1: Mathematical Modelling</b>  |   |   |   |
| Students should be able to:  |   |   |   |
| Research the background to a problem to analyse factors or variables that affect the situation.<br>Determine information relevant to the problem.<br>Decompose problems into representative parts.<br>Determine what assumptions are necessary to simplify the problem situation.  | Use abstraction to describe situations and to explain the relationship between objects and parts.<br>Identify the knowledge needed to build a mathematical model.<br>Translate the information in the problem together with the assumptions into a mathematical model that can be solved.   | Complete a solution using appropriate mathematics.<br>Create a mathematical model that can be interpreted by a computer.<br>Use computational technology to solve problems.<br>Solve the mathematical problems stated in the model.   | Analyse and perform operations in the model.<br>Interpret the mathematical solution in terms of the original situation.<br>Refine a model and use it to predict a better solution to the problem, based on previous.<br>Communicate solution processes in a written report.   |
| <b>Strand 2: Mathematical Modelling with networks and graphs</b>   |   |   |   |
| Students should be able to:  |   |   |   |
| Represent real-world situations in the form of a network.<br>Use and apply the following network terminology: vertices / nodes, edges/ arcs, weight, path, cycle.<br>Distinguish between connected and disconnected graphs, and between directed and undirected graphs.<br>Represent a graph using an adjacency matrix, and reconstruct a graph from its adjacency matrix.   | Perform calculations of square matrices by hand, with the help of a computer for large matrices.<br>Interpret the product of adjacency matrices.<br>Translate between multiple representations of mathematical ideas.<br>Demonstrate an understanding of the concepts of tree, spanning tree, minimum spanning tree or appropriate contexts.<br>Use appropriate algorithms to find minimum spanning trees.  | Use and apply dynamic programming technology, such as trees, state, optimal paths.<br>Apply Dijkstra's Principle of Optimality to find the shortest paths in a weighted directed acyclic network, and to be able to terminate the process appropriately.<br>Apply Dijkstra's algorithm to find the shortest paths in a weighted undirected and directed network.<br>Evaluate different techniques for solving shortest-path problems.   | Use algorithms to solve problems.<br>Distinguish between those algorithms which are greedy and those which use dynamic programming.<br>Justify the use of algorithms in terms of correctness and time-ability to find an optimal solution.<br>Apply network concepts to project scheduling.<br>Apply the concepts of critical path, early times, late times and float to project scheduling.  |
| <b>Strand 3: Mathematical Modelling the physical world; kinematics and dynamics</b>  |   |   |   |
| Students should be able to:  |   |   |   |
| Describe the motion of a particle in 1D (in a straight line) using words, diagrams, numbers, graphs and equations.<br>Interpret velocity and acceleration as derivatives.<br>Translate the function describing one quantity (displacement, velocity, acceleration) into functions describing the other two quantities using algebra and/or calculus.<br>Solve kinematics problems involving particle motion in one dimension.<br>Derive the kinematic formulae of motion using calculus.<br>Represent displacement as a vector.<br>Apply and interpret vector algebra. | Represent vectors in terms of components along with vectors in 2D (with orthogonal directions) and in polar form.<br>Calculate and interpret the dot product of vectors.<br>Translate the function describing one quantity (displacement, velocity, acceleration) represented as vectors into functions describing the other two quantities using algebra and/or calculus.<br>Represent and apply Newton's laws in vector form.<br>Solve constant acceleration projectile motion problems involving displacement, velocity and time.<br>Define a force as a measurable quantity.<br>Draw free body force diagrams for a particle in a smooth right-hand hemisphere or inclined plane. | Find the resultant force along a plane or between planes.<br>Resolve forces on rough and smooth surfaces.<br>Solve dynamic problems involving the motion of a particle under a constant resultant force.<br>Solve dynamic problems involving particles that collide elastically and inelastically.<br>Solve dynamic problems involving connected masses.<br>Determine the motion of a particle on a smooth rough horizontal or inclined plane under a constant resultant force. | Solve dynamic problems on rough and smooth surfaces, solve dynamic problems involving resistive forces that are proportional to $gv$ or $v^2$ .<br>Describe gravitational potential energy & how it relates to work done.<br>Describe kinetic energy & how it relates to work done.<br>Solve dynamic problems involving work done by a constant force.<br>Solve dynamic problems involving the conservation of energy for variable conservative forces.<br>Solve problems involving the dynamics of a particle moving in a horizontal or vertical circle.<br>Determine and calculate whether an object is reasonable by comparing the dimensions. |
| <b>Strand 4: Mathematical Modelling a changing world</b>   |   |   |   |
| Students should be able to:  |   |   |   |
| Calculate the first and higher derivatives of a given sequence of numbers.<br>Identify real-world situations which can be suitably modelled by difference equations.<br>Derive difference equations for real-world phenomena involving incremental change.<br>Analyze, interpret and solve difference equations in context.  | Identify real-world situations which can be suitably modelled by differential equations.<br>Derive and interpret in context differential equations for real-world phenomena involving continuous change.  | Solve linear and non-linear difference equations.<br>Identify real-world situations which can be suitably modelled by differential equations.<br>Derive and interpret in context differential equations for real-world phenomena involving continuous change.   | Solve differential equations<br>- first order linear<br>- constant or zero which can be reduced to first order<br>Interpret the solution of differential equations in context.  |

# Planning Using A Concepts Through Modelling Approach

Plan a Unit of Work for **any topic** using a *Concepts through Modelling* approach.



Refer to the Learning Outcomes Glance Card.

| Topic | Learning Outcomes | Prior Knowledge & Classwork | Links & Cross-Curricular | Success Criteria & Assessment | Real-life applications & Resources | Teaching & Learning Approaches | Start Date<br>End Date |
|-------|-------------------|-----------------------------|--------------------------|-------------------------------|------------------------------------|--------------------------------|------------------------|
|-------|-------------------|-----------------------------|--------------------------|-------------------------------|------------------------------------|--------------------------------|------------------------|

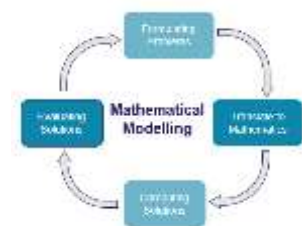


# Poster Walk



# Feedback from Groups

- How might your planning promote the making of connections between various learning outcomes.
- With a real-life application that you have selected, how might your students engage with the various stages of the Modelling Cycle.



# Reflection

What were your key takeaways from this session?

What role might a **Concepts through Modelling** approach play in your planning for teaching the Applied Maths specification?



How can you implement ideas from this session into your teaching?

